UNITED STATES AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Air Force Institute of Technology (AFIT-EN)
Wright-Patterson AFB, Ohio 45433

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

A Lanchester Model for Air Battles

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

AFIT Thesis

5. AUTHOR(S) (First name, middle initial, last name)

John H. Latchaw
Captain USAF

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
February 1972	78	29

8a. CONTRACT OR GRANT NO.

N/A

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

GSA/SM/72-8

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

Approved for public release; IAW AFR 190-17
KEITH A. WILLIAMS, 1st Lt, USAF
Assistant Director of Information, AFIT

12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

A historical verification, comparing data with model predictions, was made between the results of three World War-II bombing missions and the outcome which was obtained by allowing an analytical model to "replay" the battles. The model used to predict bomber force size as a function of time was a closed form solution for a set of differential equations which correspond to Frederick Lanchester's Square Law of combat. An attempt to measure the quality of the model was made by arbitrarily considering a prediction to be "accurate" if it was within two per cent of the known force size value.

The initial comparison results did not fall into the "accurate" category but successive improvements were made by considering two variations. The first change involved the assumption that the act of a bomber repelling an attacker had a detrimental effect on fighter aircraft effectiveness. The second alteration incorporated the assumption that the effectiveness of both bomber and fighter aircraft varied throughout the battle. This second modification produced predictions which were "accurate" during 90% of a given engagement.

Finally, a battle which included a relatively large number of bomber losses caused by ground fire was studied. A variation of the original model was used to "replay" this mission to illustrate the flexibility of the model.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Lanchester Equations						
Air Battle Model						
Historical Verification						

Security Classification

A LANCHESTER MODEL FOR AIR BATTLES

THESIS

GSA/SM/72-8

John H. Latchaw
Captain USAF

Approved for public release; distribution unlimited.

A LANCHESTER MODEL FOR AIR BATTLES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

John H. Latchaw
Captain USAF

Graduate Systems Analysis

February 1972

Preface

This thesis is the result of my attempt to evaluate how well a model which is based on a set of Lanchester equations can simulate aircraft combat losses. The method used compares predictions made by the model with force sizes which were recorded by historians. The scope of the effort was limited by considering only one type of aerial battle; bombers being attacked by fighters. A brief review of Lanchester's work is included and it is assumed that the reader is acquainted with differential equation solution techniques.

The graphs which are included to display the results of various comparisons were drawn using a CALCOMP Plotter with computations performed on a CDC 6600 digital computer.

I would like to express my appreciation for the assistance which was given by my advisor Dr. Hermann Enzer. I am also grateful for advice and encouragement received from colleagues and members of the faculty and accept sole responsibility for any errors contained in the thesis.

John H. Latchaw

Contents

List of Figures

List of Tables

Abstract

A historical verification, comparing data with model predictions, was made between the results of three World War II bombing missions and the outcome which was obtained by allowing an analytical model to "replay" the battles. The model used to predict bomber force size as a function of time was a closed form solution for a set of differential equations which correspond to Frederick Lanchester's Square Law of combat. An attempt to measure the quality of the model was made by arbitrarily considering a prediction to be "accurate" if it was within two per cent of the known force size value.

The intial comparison results did not fall into the "accurate" category but successive improvements were made by considering two variations. The first change involved the assumption that the act of a bomber repelling an attacker had a detrimental effect on fighter aircraft effectiveness. The second alteration incorporated the assumption that the effectiveness of both bomber and fighter aircraft varied throughout the battle. This second modification produced predictions which were "accurate" during 90% of a given engagement.

Finally, a battle which included a relatively large number of bomber losses caused by ground fire was studied. A variation of the original model was used to "replay" this mission to illustrate the flexibility of the model.

A LANCHESTER MODEL FOR AIR BATTLES

I. Introduction

In 1943, General H. H. Arnold commented on the planning of strategic bombing missions in a communique to Lieutenant General Carl Spaatz and Major General Ira C. Eaker.

> We know that the selection of the most vital targets must come as a result of thorough analysis. We know that the strength of our striking force will always be relatively limited. We must, therefore, apply it to those specially selected and vital targets which will give the greatest return. We cannot afford to apply it where, or in such a manner that, the return is not eminently worth the cost (Ref 19:vii).

The need for what Arnold describes as thorough analysis has long been acknowledged by military strategists, but advancements which have provided the capability to perform this analysis have been relatively recent developments. Bombing target selection and similar air combat problems of World War II generated an effort to study the various aspects of aerial warfare. Included in this effort was the development of many models of air battles.

Arnold's dilemma might have been eased had an aircraft attrition model, a model which predicts aircraft losses, been at his disposal. An estimate of expected aircraft losses provides a planner with information directly affecting the mission "cost", with an approximate number of bombers expected to reach the target which

affects the mission "return", and with an assessment of the toll which could be placed on his "limited" resources. The advantage of having this information during target selection and mission planning is apparent but the extent of this advantage is dependent upon the accuracy of the loss estimate. This example illustrates both the potential of air battle models and the need for some evaluation of their accuracy.

The purpose of this research is to accomplish a historical verification of a given air battle model and, in so doing, make an evaluation of the accuracy of the predictions which are made by the model. The model which is selected for study purports to simulate air battles which involve bomber versus fighter aircraft. Data are collected concerning bombing missions which were flown by American units during the Second World War. A Lanchester model in the form of a solution to a set of differential equations is given historical force sizes and then allowed to "refly" the missions, and the losses which the model predicts are compared with those which were recorded by historians.

The model predictions are graphed along with actual force sizes to provide a visual display of the results; a convention that is consistent with existing verification summaries with which this author is familiar. In addition, an attempt is made to place a quantitative measure on a model's accuracy by observing the errors between model and battle outcomes as a percentage of the latter. Since no

standard exists in the literature, an arbitrary criterion is adopted. Models having errors of less than two per cent are considered to be accurate and acceptable.

Following the comparison, two changes are introduced to illustrate how the model's accuracy may be improved in terms of reducing the differences between predictions and historical outcomes. First, the data that reflect the number of fighter aircraft which was destroyed are replaced with data that reflect how many of these planes left the battle for any cause and, second, the measure of skill with which airmen fire on their opponents is considered to vary throughout the battle as opposed to being constant. Each of these alterations improves the model predictions.

Finally, to illustrate the model's flexibility, the effect of anti-aircraft weapons is incorporated into the equations and predictions are compared with data from a battle which involved more than the normal amount of ground fire.

II. Lanchester Theory

Introduction

Man displays a natural tendency to invest those resources which he does possess in a manner which will prove to be rewarding. Efforts to optimize, in the sense of attaining the greatest reward, have evolved and are embodied in the science, or art, of resource allocation. Rather than optimizing, however, men who find themselves engaged in war are strongly motivated to seek acceptable rewards in return for minimum investments. In battle, the timing and size of resource commitments not only contribute to the outcome of an engagement but also affect the availability of resources which may be needed for the next conflict.

In order to enhance the probability of making a correct investment decision, certain information is desirable prior to making a selection. For instance, given a military objective, how can resources best be utilized to achieve the greatest reward? In warfare, manpower and the mechanical implements of war may be categorized as resources, while the reward may be thought of as military victory.

One trend which may be observed in historical battle outcomes is that victory is often associated with a numerical advantage. That is, an army which has greater resources than an opponent is usually victorious. This concept was not formalized quantitatively until 1916 when Frederick Lanchester published his <u>Aircraft in Warfare</u>. In this

book, Lanchester sought to answer questions involving a military conflict between comparable forces. In what way do force sizes effect battle outcomes? What losses may be expected? How long will it take to achieve victory? What force size must be committed to battle in order to assure victory?

Lanchester's work and extensions of his results have been nominated for use as attrition models in studies of conventional land, naval, and aerial battles and guerrilla warfare. However, little work has been done in verifying the theory; and this research, which considers the validity of using Lanchester's formulations to describe aerial warfare, is intended to increase the existing number of verifications.

Lanchester's Theory

Lanchester was a talented engineer-mathematician with an avid interest in the fledgling airplane and its potential as an instrument of war. He felt that whatever might be accomplished by land armies could be executed "as well or better by a squad or fleet of aeronautical machines" (Ref 18:2136). His foresight is evident by considering his prediction that "the number of flying machines eventually to be utilized by any of the great military powers will be counted not by hundreds but by thousands, and possibly by tens of thousands, and the issue of any great battle will be definitely determined by the efficiency of the

aeronautical forces" (Ref 18:2136). Lanchester's work led him to a mathematical analysis of the relations between opposing forces in battle. The end results of this analysis revolves about his conclusion that a numerically superior force can always achieve a victory, providing that opposing weapons systems are equitable. Under battle conditions which were considered to be "state of the art" for Lanchester's time, he developed an expression that translates a numerical advantage into a military advantage. Specifically, the effective strength of a force is proportional to the square of the number of combatants entering an engagement. For instance, a force which outnumbered an opposing force at a ratio of three to two would have the advantage of being able to destroy enemy fighting elements more effectively; and this superiority could be measured as a ratio of nine to four. This relationship is called Lanchester's Square Law.

Lanchester called the applicable conditions for which the Square Law holds, "the conditions of modern warfare", which essentially allow combatants to select from a number of targets and to attack with continuous fire. An example of a confrontation which conforms to the Square Law assumptions would be opposing fronts of infantrymen armed with rifles.

Battle conditions which involve one-on-one encounters are termed "ancient combat" by Lanchester. In such battles, the effective strength of a force is directly proportional

to the number of combatants entering the engagement. This relationship is known as Lanchester's Linear Law and in these cases the ratio of force sizes may be used as a measure of military advantage. The Linear Law applies to present day conflicts in which combatants direct their fire towards an area rather than an individual target. A siege in which an attacking force attempts to overpower a force defending a fortified position is a situation in which the Linear Law may be applied. In this type of engagement, it is not uncommon for the aggressor to bombard the fortified position with area directed fire before initiating an assault.

Lanchester Equations

Lanchester expresses his theory in mathematical statements by considering how force size varies during the course of battle. To accomplish this, he represents force size as a function of time and assumes that the battle is continuous and terminates with the annihilation of one force. Let m represent one force size at time t and n represent the opposing force size at time t. Let A be a parameter representing force m's combat loss rate per opposing combatant. That is, A expresses the number of m's fighting units which a member of the n-force is capable of destroying per unit of time.

The parameter A is most frequently interpreted as the product of the rate of fire of a single n-force

fighting element and the probability of destroying an m-force element with a single round of fire. Let B similarly represent force n's combat loss rate. The dot notation shall be used here and throughout the thesis to denote differentiation with respect to time. Using the preceding conventions, a battle conforming to the Square Law conditions may be represented by the following set of differential equations.

$$\dot{m} = -An \qquad (1)$$

$$\dot{n} = -Bm \qquad (2)$$

A solution for this set of equations can be obtained in the following manner. Differentiate and rearrange equation (1).

$$\ddot{m} + A\dot{n} = 0 \qquad (3)$$

Substitute -Bm from equation (2) for $\dot{n}$ in equation (3).

$$\ddot{m} - ABm = 0 \qquad (4)$$

Solve equation (4) and let y represent $\sqrt{AB}$.

$$m = c_1 e^{yt} + c_2 e^{-yt} \qquad (5)$$

Similarly

$$n = k_1 e^{yt} + k_2 e^{-yt} \qquad (6)$$

Let the initial force sizes at time zero be denoted by m_0 for the m-force and by n_0 for the n-force. Substitute zero for t in equations (5) and (6).

$$m_o = c_1 + c_2 \tag{7}$$

$$n_o = k_1 + k_2 \tag{8}$$

Differentiate equation (5) and substitute the resulting expression for $\dot{m}$ in equation (1). Also substitute the right hand portion of equation (6) for n in equation (1).

$$c_1 y e^{yt} - c_2 y e^{-yt} = -A(k_1 e^{yt} + k_2 e^{-yt}) \tag{9}$$

When t equals zero, equation (9) becomes

$$y(c_1 - c_2) = -A(k_1 + k_2) \tag{10}$$

Similar substitutions from the derivative of equation (6) and equation (5) in (2) produce

$$y(k_1 - k_2) = -B(c_1 + c_2) \tag{11}$$

Using equations (10) and (8), one obtains

$$n_o = k_1 + k_2 = \frac{-y}{A}(c_1 - c_2) \tag{12}$$

Solving equations (7) and (12) simultaneously results in expressions for c_1 and c_2,

$$c_1 = \frac{m_o - n_o(A/y)}{2} \tag{13}$$

$$c_2 = \frac{m_o + n_o(A/y)}{2} \tag{14}$$

Using equations (11) and (7), one obtains

$$m_o = c_1 + c_2 = \frac{-y}{B}(k_1 - k_2) \tag{15}$$

Solving equations (8) and (15) simultaneously results in expressions for k_1 and k_2,

$$k_1 = \frac{n_o - m_o(B/y)}{2} \tag{16}$$

$$k_2 = \frac{n_o + m_o(B/y)}{2} \tag{17}$$

Replacing y with $\sqrt{AB}$ in equations (13), (14), (16) and (17) and substituting these expressions for the constants in equations (5) and (6) gives the desired results.

$$m = \left(\frac{m_o - n_o\sqrt{A/B}}{2}\right) e^{yt} + \left(\frac{m_o + n_o\sqrt{A/B}}{2}\right) e^{-yt} \tag{18}$$

$$n = \left(\frac{n_o - m_o\sqrt{B/A}}{2}\right) e^{yt} + \left(\frac{n_o + m_o\sqrt{B/A}}{2}\right) e^{-yt} \tag{19}$$

Rearranging terms yields

$$m = m_o\left(\frac{e^{yt} + e^{-yt}}{2}\right) - n_o\sqrt{A/B}\left(\frac{e^{yt} - e^{-yt}}{2}\right) \tag{20}$$

$$n = n_o\left(\frac{e^{yt} + e^{-yt}}{2}\right) - m_o\sqrt{B/A}\left(\frac{e^{yt} - e^{-yt}}{2}\right) \tag{21}$$

Or $m = m_o \cosh yt - n_o\sqrt{A/B} \sinh yt$ (22)

$$n = n_o \cosh yt - m_o\sqrt{B/A} \sinh yt \tag{23}$$

The ratio of A to B is termed the exchange rate and may be used to indicate which of the opposing forces has

the greater effectiveness. Let E represent this exchange rate. A value of E greater than one would indicate that the n-force has a superior edge in killing effectiveness, whereas a value less than one would indicate that the m-force is superior in this respect. Let m_0 and n_0 represent the initial size of the m-force and n-force respectively. A general solution for equations (1) and (2) yields an expression for the exchange rate.

$$(m_o^2 - m^2) = E(n_o^2 - n^2) \tag{24}$$

When the opposing forces are quite similar in fighting elements and skills, then the corresponding loss rates may be considered equal. This is the case when two armies equipped with comparable weapons systems engage in conflict. Suppose that each force's loss rate is one tenth per unit of time (A = .10; B = .10). Then, the exchange rate equals one and the outcome is determined by the size of the forces which participate. For example, suppose the m-force enters the battle with 1,400 armed men and the n-force begins with 1,000. Then the n-force will be annihilated and the m-force will have approximately 980 survivors. These results may be obtained by setting n equal to zero and solving equation (24) for the correct value of m. The duration of the battle may be determined by setting equation (23) equal to zero and solving for t or, similarly, by setting equation (22) equal to 980 and solving for t. In the example, t is approximately °.959

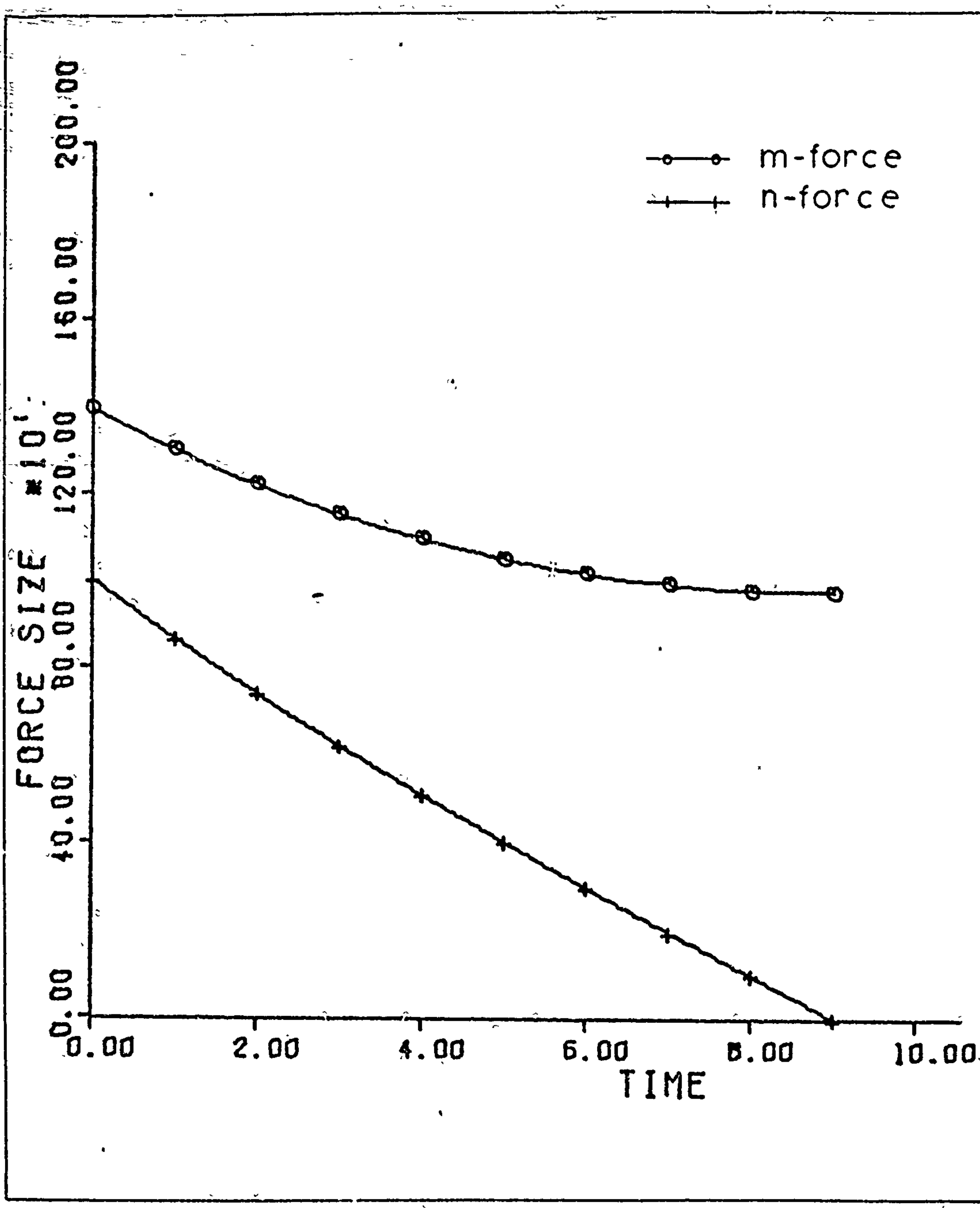

Figure 1 An Example of Lanchester Model Predictions for a Military Engagement

units of time. Equations (22) and (23) might also be used to predict respective force sizes for a given value of time. A graphical representation of this sample battle is shown in Figure 1.

If either force possesses an advantage in skill or equipment, then the exchange rate may be such that a numerically inferior force can achieve victory. Suppose, in the example above, that the combat loss rate for the m-force is one tenth and that the n-force combat loss rate is one twentieth ($A = .10$; $B = .05$). Under these conditions, the n-force can achieve victory and count approximately 140 survivors.

A battle which conforms to the Linear Law conditions may be represented as shown by equations (25), (26) and (27). Here the combat loss rate A, may be interpreted as the Square Law parameter times the ratio of the average area presented by an m-force element and the total area over which the fire of the n-force is directed.

$$\dot{m} = -Amn \tag{25}$$

$$\dot{n} = -Bmn \tag{26}$$

$$(m_0 - m) = E(n_0 - n) \tag{27}$$

When fire is directed toward an area, there is no destructive effect unless a vulnerable portion of the area is struck. During a siege in which the fighting elements are infantrymen, a member of the entrenched force will

barricade himself to minimize his exposure to fire. The area a man presents as a vulnerable target could be approximately one square foot. Let m denote the size of the entrenched force and n denote the size of the attacking force. Let r_n be the rate of fire of an n-force, P_n be the single shot kill probability for an n-force element, A_m be the average area presented by an m-force element and A_{tm} be the total area that is occupied by the m-force. Equation (25) may be rewritten to display these factors which determine the value of the parameter, A.

$$\dot{m} = (r_n P_n n)\left(\frac{A_m m}{A_{tm}}\right) \qquad (28)$$

The change in size for the m-force in this, the Linear Law application, is equal to the change which would be experienced in a Square Law engagement times the proportion of the occupied area which is vulnerable. As one would expect, the Linear Law losses occur less rapidly than do losses for a Square Law battle involving the same forces.

It should be noted that it is assumed that m-force elements are evenly distributed about their fortified area. If they are clustered and this is known to the n-force, the attackers' fire will be concentrated toward this cluster. The effect of such an action will be a reduction of A_{tm} and an increase in the m-force loss rate.

Generalization

The elementary Square Law equations can be extended to account for reinforcements during an engagement and for losses which are not the direct result of active combat. Such additional losses may be termed operational losses. Let C and D represent the operational loss rate for the m-force and n-force respectively. Let P and Q represent the reinforcement rate for the m-force and n-force as a function of time. The set of differential equations for a Square Law formulation incorporating these extensions is

$$\dot{m} = P - An - Cm \tag{29}$$

$$\dot{n} = Q - Bm - Dn \tag{30}$$

The generalization of equations (1) and (2) represents an attempt to measure force size changes by considering three contributing factors. Operational loss rates are frequently based on historical trends and are measured per fighting element. That is, losses which are attributed to illness, accident, desertion, etc., tend to occur at a constant rate and are included in the equations as the product of an observed rate and the current force size.

Reinforcements tend to occur when there exists a need and available resources allow their implementation. Hence, they occur sporadically and the reinforcement rates are frequently represented by functions which are specified for time intervals. For example, a reinforcement rate function

may be assigned a value of the number of reinforcements for the unit of time during which the additional fighting elements arrive and assigned a value of zero for all other times. This concept involves the assumption that reinforcements arrive at a constant rate during each unit of time. Suppose that the unit of time is one day and that the m-force receives 3,000 reinforcements on day two and 1,200 more on the fifth day of conflict. The reinforcement rate P, can be represented as the step function

$$P = \begin{cases} 0, & 0 \le t < 2 \\ 3{,}000, & 2 \le t < 3 \\ 0, & 3 \le t < 5 \\ 1{,}200, & 5 \le t < 6 \\ 0, & \text{all other} \end{cases} \tag{31}$$

Lanchester Related Studies

Since the publication of Aircraft in Warfare, a number of analysts have presented extensions and variations of Lanchester's theory. Included in these efforts are examinations of the mathematics which are involved, refinements of the original equations, specializations for application to particular combat situations, estimates of parameter values and a few verifications. A rather extensive account of existing works may be found in Frick's paper, "Interaction of Forces as Discrete Processes" (Ref 12:9-25).

Assumptions

Many of the studies which have appeared over the past 55 years include attempts to apply Lanchester's theory to

different types of conflict. The battle conditions which must hold in order that these applications be realistic are described through the assumptions which accompany a theory. The following seven assumptions are associated with Square Law engagements by the United States Air Force Assistant Chief of Staff, Studies and Analysis (Ref 25:29). An annotation is included to discuss the implication of each assumption.

1. Each force is composed of many elements, so that statistical fluctuations are unimportant. The vague term "many" makes this the most flexible assumption. Lanchester equations have been applied to land battles involving thousands of men as well as to naval conflicts involving less than fifty vessels indicating that the range which is interpreted to include many is extensive. Air battles involving more than one hundred elements of each force are felt to readily conform to this assumption.

2. Each force is homogeneous. The fact that airplanes are usually designed and constructed with some particular performance capability in mind allows a fleet of aircraft to be considered homogeneous in the sense that each element performs the same function. For instance, one type of aircraft may possess the capacity and the durability to transport weapons over long distances and deliver them on ground targets. Another type may be endowed with sufficient speed and maneuverability to be utilized in air-to-air combat. When a force is composed

of one basic type of aircraft such as a fleet of bombers, it can be assumed to be homogeneous. While this study is limited to such forces, the problem of heterogeneous forces has been considered and, to an extent, circumvented in other works. O. Helmer's "Combat Between Heterogeneous Forces" (RM-6) and R. N. Snow's "Contributions to Lanchester Attrition Theory" (RA-15078) are two examples of contributions which deal with heterogeneous forces.

3. The location of the forces is ignored. The differences between an air battle conducted over London and one over Berlin are small. Since aircraft operate in a medium which has some stable characteristics the world over, the location of the conflict would seem to be of little or no consequence. However, if one force's elements have a greater fuel capacity than the others, then the location of the battle in relationship to the latter force's air bases may have a definite effect on the time spent in refueling and the time spent in actual battle. Time which is used to refuel and to travel to and from the battle area obviously cannot be used for destroying enemy aircraft. If the battle is moving with respect to the positions of these refueling bases, then the effect of this disadvantage will vary with time. Thus, the degree to which location affects a battle outcome can either be neglected or must be considered as an important factor but is in any event difficult to measure. Whenever the fuel capacity disparity exists, acceptance of this

assumption may be interpreted as a simplification of reality.

4. All elements of the forces are subject to fire from all enemy elements. As compared to terrestrial warfare, the three-dimensional freedom of movement which an aircraft enjoys tends to improve an element's ability to select and fire on any element of the opposing force. If one force's mobility is restrained by the requirement to maintain a flying formation with other force members, then the opposing aircraft may have some advantage in maneuverability. Target selection might also be restricted by an element's ability to locate enemy aircraft, but this factor is minimized when the planes are equipped with radar or other electronic sensor devices.

5. The firing rate of the forces is limited by technical capabilities and not by logistics or tactical decisions. Firing rates for aircraft are determined by the armament aboard, whether guns are fixed or can be rotated, the intervals during which targets are within range, and the amount of ammunition which can be carried. All of these factors may be described as technical in nature, and it is assumed that the only tactical factor involved is that individuals will fire their weapons in earnest at every opportunity. Logistics must be considered whenever fuel limitations are involved. Naturally, when an aircraft breaks off from the battle to refuel, the firing rate of that plane drops to zero. This alone may be considered a

basis for concluding that some air battles do not conform to this assumption, but this may be a minor argument as all fighting elements have similar limitations. Soldiers become physically and mentally exhausted, for instance. It is just that in an air battle, the abrupt change in firing rate is more dramatic since an aircraft without fuel is, of necessity, eliminated from the battle. Air battles meet this assumption to the same extent as do other forms of conventional warfare.

6. An element is either destroyed or completely undamaged. Aircraft have the capability to fly and perform after sustaining structural damage; and, while a damaged plane cannot perform as well as an undamaged one, it is obvious that during the course of an air battle some planes can be categorized as neither destroyed nor completely undamaged. This assumption may be accepted as a simplification or may be modified by adjusting the two categories.

7. Fire is aimed at an element and elements not aimed at are not damaged. This assumption is directly applicable to air battles as from the time of opening fire, the individual aircraft is the mark of the gunner.

Modification of these assumptions for use with Linear Law engagements requires only one alteration. The final assumption must be changed in the following manner: fire is directed at an area in which opposing forces are believed to be located; elements not located in such an area are undamaged.

III. Model Verification

Because the term "model" is used in many contexts, one definition is chosen for use in this thesis. The selection is attributed to RAND Corporation's E. S. Quade who describes a model as "a simplified representation of the real world which abstracts the features of the situation relevant to the questions being studied" (Ref 10:2). Some hint of model flexibility should be added to this definition since a good model must be capable of being perturbed in order to cope with real world fluctuations of those features which are important. A solution for a set of Lanchester equations is a model in the sense that it represents a real world military engagement and abstracts from the nature of combat losses that are suffered on either side throughout a conflict. The times when losses occur and the size of these losses are considered to be features relevant to the study of battles.

Need for Verification

A model is frequently used as an aid in decision-making and, to this end, a model's value is closely related to the level of confidence which the decision-maker places in it. One method of gaining confidence in a model is to demonstrate its past record of prediction accuracy by means of historical validations. If a model's predictions correspond closely to recorded observations, and, if deviations can be plausibly explained, then the model may be considered

to be more attractive than an untested one.

Included are three opinions on the need for verification. In a historical validation prepared for the Center for Naval Analyses, William W. Fain expressed these views:

> Today, many models are actually being used to aid real decision-making. By and large these models are purely subjective judgements of the people who were involved in their development and use. That is, they are virtually untested against anything but a criterion of "reasonableness".
>
> If these models are false, they can of course lead to false answers and false decisions--this could be very costly to the nation in money, lives, and even the future of the country.
>
> The time has come, I believe, to divert at least a portion of the effort devoted to model development and use to an effort to validate the models we have (Ref 10:18).

A well-known authority on Lanchester's theory, Joseph Engel of the University of Illinois, put it more succinctly by saying "an operational research scientist must seek validation of his mathematical models" (Ref 26:192). In an assessment of contributions to the study of Lanchester's theory, Ladislav Dolansky comments: "The number of cases that have been studied by means of the Lanchester approach and verified by means of actually observed data is extremely small. Therefore, the utility of this approach with respect to a good prediction in a particular case remains to be established by means of additional verification studies" (Ref 7:351).

Recognizing the need for verifying existing models is, of course, not an end in and of itself. Once verifications

are attempted, some method of interpreting the results must be established. For instance, while successful validations tend to strengthen a model's reputation, isolated experiments are not sufficient to allow unqualified claims of accuracy and reliability. If collaborating results are in order, how many verifications should be required before confidence may be placed in a model? The small number of validations to which Dolansky refers have been too few to establish some standardization of the methods to be used in historical verifications and the means used to evaluate the results.

Existing Verifications

The list of extensions and variations of Lanchester's theory includes relatively few studies which involve historical validations. The prime reason for this dearth is the scarcity of historical warfare data. Even when available, data which are recorded under the stress of combat are frequently not very reliable. The most prominent of those validation attempts which do exist are worth mentioning.

The intial validation appears in Lanchester's original text. He cited the naval Battle of Trafalgar as an example of the application of a tactic known as concentration. To gain the advantage of concentration one attempts to divide an opposing force for the purpose of gaining numerical superiority. Lanchester's comparison shows that the

advantage and ultimate victory gained by the British is reflected by the predictions of his Square Law model.

A second validation was accomplished by Engel in 1954 (Ref 9). Engel compared historical battle data from the United States Marine invasion and capture of the island of Iwo Jima during World War II with predictions from a variation of the elementary Lanchester Square Law equations. He found only two parameters had to be estimated and he worked without precise knowledge of Japanese losses. Engel's results have been described by R. E. Bach (Ref 2) as being a remarkably good fit, but no criterion for measuring the agreement between model predictions and data was used.

"Validation of Combat Models Against Historical Data" by Fain (Ref 10) is similar to Engel's work. Fain's comparison was compared with land battles and data were taken from two Korean Conflict engagements. He attempted to estimate loss rate parameters from the characteristics of the weapons systems involved, in addition to estimating these values from the data, and derived results which are comparable to Engel's.

Herbert K. Weiss (Ref 27) used data taken from the American Civil War in his study and developed a model which is a variation on Lanchester's theme. D. Willard's "Lanchester as a Force in History; An Analysis of Land Battles of the Years 1618-1905" (Ref 28) is a validation which provides little support of Lanchester's theory and includes

the conclusion that military battles are too complex to be modeled by a pair of equations. However, it is not apparent that the battle conditions in this survey satisfy the basic Lanchester assumptions. Robert Helmbold's study (Ref 14) represents an effort to emphasize some characteristics which are suggested to be common to both land and air battles. He used initial and terminal force size data from the World War II Battle of Britain in an attempt to correlate the exchange rate E with initial force sizes. Helmbold's study differs from the others in that he did not attempt to predict losses for various times throughout the battle.

IV. Data

Problems

As previously mentioned, the principle deterrent to historical validation is the scarcity of data. Once data are located, the researcher must make sure that the chosen data possess an acceptable degree of accuracy. The mere nature of warfare causes data collection to become suspect. Periods of interest when loss rates are significantly larger than zero invariably correspond with intense periods of battle. One cannot be expected to record objective observations while the need for personal survival is paramount. Hence, data resulting from reports of participants are frequently based on recollections of events which transpired during a particularly hectic time. Data which result from a historical analysis of a battle have the advantage of being more objectively recorded. The researcher may choose from information recorded immediately after an event, which may be distorted because of the excited state of the observer, or from recorded reflections which are only as accurate as the memories of their contributors.

Data sources used in this study include official military histories, periodicals, personal memoirs and professional histories. Whenever a specific force size was repeated in a number of sources, its reputed accuracy was felt to be enhanced. While figures did vary in various accounts, there existed general agreement as to the number

of planes which were committed to the battles and the number of surviving aircraft.

Battle Type

Data collection progresses in one of two manners. The first involves using what is considered to be an excellent data source and then choosing a battle type which conforms to these data for study. In the second method, a battle type is selected and, subsequent to this selection, data from historical battles corresponding to this kind of conflict are reviewed. In this second case, a search is initiated for the best data available and, should data of an expected degree of accuracy not be available, the study is terminated. The second procedure was adopted for this research simply because a set of Lanchester equations for an air battle was available.

The type of battle which was selected for study shall be called the strategic bomber battle. A target which is defended by fighter aircraft is selected for bombing and a bomber force launched to accomplish this task. Once the bomber fleet is detected and its intentions confirmed, some of the fighter planes are sent to intercept it. The prime concern of the bomber aircrews is to drop their payload on the designated target. In order to do this, they must defend themselves when attacked by fighter aircraft. They do not seek and destroy interceptors and would much rather make their run undetected or unattacked. Fighter

pilots, on the other hand, have the singular objective of locating and destroying bombers before they reach the target area. For the purpose of this study, it is assumed that the battle takes place under visual conditions. That is, opposing aircraft are aware of one another's location, either through sight or electronic contact.

Selection of this battle type to be represented by Lanchester equations is acceptable only if the battle conditions conform with the Lanchester assumptions. The fighter aircraft have the freedom to attack any available bomber. In return, bombers may choose to fire on any fighter aircraft which is within range. This "within range" restriction is no more binding in an air battle than in a battle involving land armies or naval vessels.

The concept of an element either being destroyed or totally undamaged cannot be accepted literally. However, it is as realistic as any of the other assumptions once definitions of destruction are developed. For the purposes of this research, a bomber shall be considered to become a loss when it crashes to earth or lands in an enemy controlled region. A fighter aircraft shall be considered destroyed once it is unable to press the attack. It should be noted that war models are simplifications of actuality, and that the acceptance of the destroyed/undamaged concept is a reflection of this fact. The remaining assumptions for the Square Law case are considered to be met.

Explicit Data

When extensive data for a particular air battle do exist, their existence may usually be attributed to the fact that the battle was in some way distinguished. Even before entering the Second World War, the United States committed the bomber segment of its Air Corps to high-level daylight tactics. This was evidenced in equipment and aircraft design and aircrew training. The large amount of armament placed aboard the aircraft reflected the belief that the bombers were capable of self-defense. When the Combined Bomber Command was formed following the Casablanca Conference of 1942, a running debate began between British and American strategists over the respective merits of night time and daylight bombing.

The United States' Eighth Air Force mission flown on 17 August 1943 consisted of two large forces of Boeing B-17 "Flying Fortresses" which were dispatched from air bases in England against targets in deepest Germany. The lead force was assigned the Messerschmitt aircraft factories at Regensburg as targets and the second force was assigned the ball-bearing manufacturing complex located at Schweinfurt. The first force was to shuttle to North African air bases while the second group was to return to England. This mission received distinction as an acid test for America's long range daylight bombing tactics. This "testing" aspect prompted several histories of the mission to be written. The prime reason for selecting this

mission is that data are readily available and there exist enough sources to substantiate the reliability of the data.

The Schweinfurt force's takeoff was considerably delayed by weather conditions. This allowed German fighter aircraft the opportunity to refuel and rearm, an effort which was accomplished in anticipation of the Regensburg force's return. The interceptors located no targets when the lead force turned south after dropping their bombs but were well prepared to engage the tardy second fleet. This circumstance allows the mission to be divided into two separate battles. This is a second reason for its study, since it provides two data sets. A third reason for selecting this mission is that fighter escorts were forced to withdraw because of fuel limitations after crossing the German border and prior to interception. This takes care of the Lanchester assumption that the bomber force should be homogeneous in composition.

Another choice for study is the 1 August 1943 mission flown by Consolidated Vultee B-24 "Liberators" of the Eighth and Ninth Air Forces from Bengasi, Libya against oil refineries located at Ploesti, Rumania. The Ploesti raid has taken a prominent position in air warfare history as a result of two distinguishing points. First, the Ploesti mission, a result of the Casablanca Conference, was planned as a panacea effort which would both hasten the conclusion of the war and satisfy the desires of some officials for retaliation against the 7 December 1941

Japanese attack on Pearl Harbor. The second point is perhaps more important as it makes the mission a "one of a kind" effort. The flight plan called for a large force of high altitude bombers to deliver their weapons from ground level. The Ploesti mission was selected for study on the basis of data accessibility and homogeneity of forces; once again, the bombers were unescorted. In addition, the study of low level tactics has present day applicability. At this time, the United States Air Force is endorsing procurement of a strategic bomber with low level capabilities. A model which could accurately depict losses under such battle conditions would be of interest.

While these selected data sets may, at first, seem to be dated, it should be noted that the experiment concerning the value of long range daylight bombing was not considered to be a resounding success. It was generally concluded that the long range bomber could not provide a viable system until fighter escort could be provided for the duration of a mission. As a result, few unescorted missions were flown after the early autumn of 1943. This fact severely limits the number of engagements involving homogeneous forces which are available for study. Secondly, data of a classified nature were avoided during the research. This policy limited the number of engagements from later conflicts which could be studied. It is assumed that as long as the strategic bomber remains a member of

the defense triad, studies of the chosen battle type will be worthwhile.

Data Manipulation

The Regensburg and Schweinfurt missions were quite similar. Each was characterized by air-to-air confrontations which were sustained for lengthy durations as the bomber forces crossed over Germany, and, in the Schweinfurt instance, as the bombers returned to England. As the formations moved across the continent, fresh fighter elements were launched from air bases along the route to relieve those with exhausted fuel and ammunition. There exist no reports of individual bombers exhausting defense ammunition. Thus, these battles may be assumed to be continuous in a real sense.

Each battle was considered to begin with the first fighter attacks. Operational losses such as turn backs caused by mechanical failures are not considered. To adjust for this, the initial bomber force size is taken to include only those aircraft which were still on course at the beginning of the battle. Since the mission routes were designed to avoid known anti-aircraft installations, and, since John Sweetman (Ref 24) describes the anti-aircraft efforts at the time to be virtually ineffective, the damage inflicted by these weapons is ignored. Actually, three and one-third percent of the bomber losses were attributed to anti-aircraft fire. This simplification is not

considered to have a significant effect on the agreement between data and model predictions.

The times of bomber losses are computed from known locations of the losses, average ground speed and known times over various reference points. Times are normalized to allow the length of a battle to equal one in order to ease the comparison of the two battles. The battle is considered to end when the final bomber loss occurs.

Data were not available for the computation of individual fighter loss times. Fighter losses are assumed to behave in accordance with Lanchester theory to the same extent as bomber losses do. This concept is not without precedent, as may be seen from Engel's paper (Ref 9).

Certain properties of the Ploesti mission caused the course of battle to differ from the Regensburg and Schweinfurt conflicts. Planners for the mission predicted that the low-level approach would severely hamper ground radar effectiveness and anticipated that the force would not be detected until it was near the target area. In actuality, Ploesti defense forces were aware of the bomber fleet's position soon after takeoff. However, a series of human errors prevented the fighter pilots from locating the bombers until they were very near the city. Thus, the resistance at Ploesti turned out to be as anticipated and the majority of losses occurred in the target area. Known losses of individual groups were assigned times of

occurrence which coincided with the applicable unit's time over target. Times of losses which were suffered enroute to home air bases were computed in the same manner as the Regensburg and Schweinfurt times. Times were once again normalized.

The optimists among the Ploesti planners felt that the low-level attack would reduce losses to interceptors and to large (eighty-eight millimeter) anti-aircraft guns. They reasoned that the fighter's sphere of maneuverability would be reduced by half and that German artillery lacked the mobility necessary to track a two-hundred mile per hour B-24 at short range. However, these same individuals were unable to anticipate those losses which would result from smaller ground fire, secondary explosions and fires, and collisions with vertical structures. The data reveal that only twenty percent of the losses could be attributed to the efforts of interceptor aircraft. Losses to ground fire cannot be ignored in attempts to model this battle and, in this light, the Ploesti mission offers a data base which may be used to test a given model's ability to react to abnormal situations.

The basic source used for collecting data on the bomber force size for the Regensburg and Schweinfurt battles is Schweinfurt--Disaster in the Skies (Ref 24). The enemy force size and losses for these battles are found in The Mighty Eighth--Units, Men and Machines (Ref 11). These and other concurring sources are considered to represent a

reasonably accurate description of the events as they actually occurred. Locations of the bomber losses are found on maps of the mission route. The accuracy of the times of aircraft losses is sensitive to any errors in computations of the distances between crashes and the ground speed estimate which is 215 miles per hour.

Tables depicting the number and causes of bomber losses for the Ploesti mission are found in The Ploesti Mission, 1 August 1943 (Ref 19) and The 9th Air Force in World War II (Ref 20). The prime source for ascertaining times of bomber losses and fighter strength and losses is Ploesti (Ref 8). Figures from these sources are considered to be quite accurate.

Bomber formations were designed and flown to present the best defense against fighter attacks. The loss of an aircraft could conceivably weaken this defensive posture. It may therefore be quite possible that one bomber loss contributed to the loss of others, indicating that such losses were not independent events. Many fighter pilots favored attacking bombers which flew in the low, rear position. This tactic forced a bomber's probability of survival to be somewhat dependent upon the position that it flew in the formation. This implies that the probability of becoming a casualty was not the same for all aircraft. The measure of the sum of the absolute values of errors was used to compare different model results under the assumption

that the effect that bomber loss interdependence and varying survival probability had on the errors was constant for each model.

V. Methodology

Basic Model

The basic model which is chosen to represent the strategic bomber type of air battle is taken from Frick's paper (Ref 11:32). Frick reasons that the bomber losses are related to the number of attacks made on the bomber force since the greater the number of attacks, the greater the number of rounds which may be fired. In a Lanchester model the number of rounds fired throughout the battle may, of course, be reflected in the combat loss rate. The number of attacks which a fighter force can make is dependent upon the fighter force size and the length of the battle. The bomber loss rate may be represented by a differential equation similar to those which appear in the elementary Lanchester Square Law formulation. Secondly, bombers are considered to fire at the fighters only when under attack. This implies that the number of bombers which are firing at any time is no greater than the number of fighters and the equation which represents the fighter loss involves the fighter force size rather than the bomber force size. The selected model is a slight variation of Lanchester's Square Law model.

Using these observations, the air battle type which is studied may be readily represented by the following equations. Let m represent the bomber force size at time t and I represent the bomber combat loss rate. Similarly, let

n represent the interceptor force size at time t and B represent the fighter combat loss rate. Then,

$$\dot{m} = -In \tag{32}$$

$$\dot{n} = -Bn \tag{33}$$

To effect the comparison between this model's predictions and bomber losses which are recorded in the historical data, the differential equations must be solved simultaneously to yield an expression for m. Equation (33) may be solved as a linear differential equation,

$$n = k e^{-Bt} \tag{34}$$

By assigning t the value of zero, k may be determined to be equal to n_o, where n_o represents the fighter force size at the beginning of the battle. Substituting this solution of (33) into (32) yields the following expression for m via direct integration.

$$m = \frac{I}{B} n_o e^{-Bt} + k^* \tag{35}$$

Setting t equal to zero, solving for k* and adopting the m_o notation to represent the initial bomber force size gives,

$$m = m_o - n_o \frac{I}{B}(1 - e^{-Bt}) \tag{36}$$

This bomber force size function may be plotted, bombers against time, provided the two parameter values I and B are known or can be rationally estimated. If equation

(34) is substituted in (36), the relation between fighter force and bomber force is given by the equation

$$(m_o - m) = \frac{I}{B}(n_o - n) \tag{37}$$

Although the model is constructed using Square Law assumptions, the nature of the equations forces the exchange rate relationship to resemble that of the Linear Law. For the purposes of this research, this model shall be referred to as the Linear Model. The only known value for n other than n_0 is n(t) for t equal to one. Hence, the exchange rate I/B = E may be estimated by the ratio of the total change in bomber force size to the total change in fighter force size and may be computed by using equation (37). The initial force values, m_0 and n_0, and the terminal force sizes are known from available data. Since times were normalized to allow the end of the battles to occur when t equals one, let m(1) and n(1) represent the terminal force sizes for the respective bomber and fighter fleets. Equation (37) may then be rewritten.

$$E = \frac{m_o - m(1)}{n_o - n(1)} \tag{38}$$

Once this exchange rate is estimated, a value may be assigned to B by letting m equal the historical number of surviving bombers at the end of the battle, or, when t equals one. Substituting m(1) in equation (36) and solving for B will give the result

$$B = \ln\left(\frac{E\,n_o}{m(1) - m_o + E\,n_o}\right) \tag{39}$$

An optional and equal evaluation of B may be obtained by substituting the known terminal value for fighters, n(1), in equation (34).

$$B = -\ln\left(\frac{n(1)}{n_o}\right). \tag{40}$$

These calculations of E and B values in turn determine the value of the parameter I. The comparison between model predictions and data values is then confined to points located between the battle beginning and end. This method is also employed in the verifications undertaken by Engel and Fain.

Three steps were taken to allow the comparison of model predictions and historical data and, eventually, the comparison between various models. A scatter diagram of data values and an m-curve of bomber force values is drawn on the same plane for each battle for the purpose of visual comparison. This method of presenting results is the primary one used in existing verifications. The sum of the absolute values of errors is computed for each battle for the purpose of comparing different models. This value is referred to as the error sum in this thesis and it is assumed that a change in a model which results in a reduction of the error sum constitutes an improvement in that model.

A percentage error, computed to be the absolute value of an error divided by the historical force size, is determined for each applicable data point (0<t<1) for the purpose of describing a "goodness of fit" and the criterion of a 2% error is adopted. Percentage errors provide a numerical measure which may be used in assessing a model's accuracy and in comparing two or more models. Should a model that always makes predictions that are within 2% of actual values be adopted for use? Is 2% too demanding or too lenient a criterion? Is a model with 2% errors always preferable to a model with 3% errors? Answers to questions such as these depend on the intended use of the model, available alternatives, and preferences of the decision-maker. While the selection of the value 2% is quite arbitrary, it is included to emphasize this writer's belief that a quantitative measure of verification results is desirable.

The value below which nine-tenths of the percentage errors fall for any given battle is also noted. This value, referred to as the ninety percent bound, reflects how well the model predicts most of the time. For instance, suppose a model is used to predict losses for a battle and comparison of these numbers with historical data reveals that the largest percentage error is 3.7% and the ninety percent bound is 1.8%. These measurements show that the model meets the criterion throughout most of the battle and that the large percentage errors occur

infrequently. It is possible that these errors might be attributed to some other factor than the model itself; perhaps to data collection. The ninety percent bound may also be used for comparing models. Suppose model A has a greatest percentage error of 1.9% and model B has one of 1.75%. Model B would be favored based on these measurements but the selection might be reversed if model A has a smaller ninety percent bound; for instance, 1.67% for A compared to 1.72% for B.

Graphs for the Regensburg and Schweinfurt missions are shown in Figures 2 and 3 respectively. On these, and on other graphs throughout the thesis, the solid curve represents model predictions and the crosses represent recorded historical force sizes. In these two cases, the exchange rate was estimated by using total changes in fighter force, which shall be referred to as the actual number of fighters destroyed. This data value represents the number of fighter aircraft which were damaged beyond repair on the day of the battle. Such data are taken from post mission intelligence reports or post war examination of German records and are considered to be conservative. The exchange rate superiority belongs to the fighter forces in these battles. The near linearity of the bomber curve reflects the fact that, should the battle have continued indefinitely, the bomber force would have been annihilated and the fighter force would then have achieved victory in a true Lanchester sense.

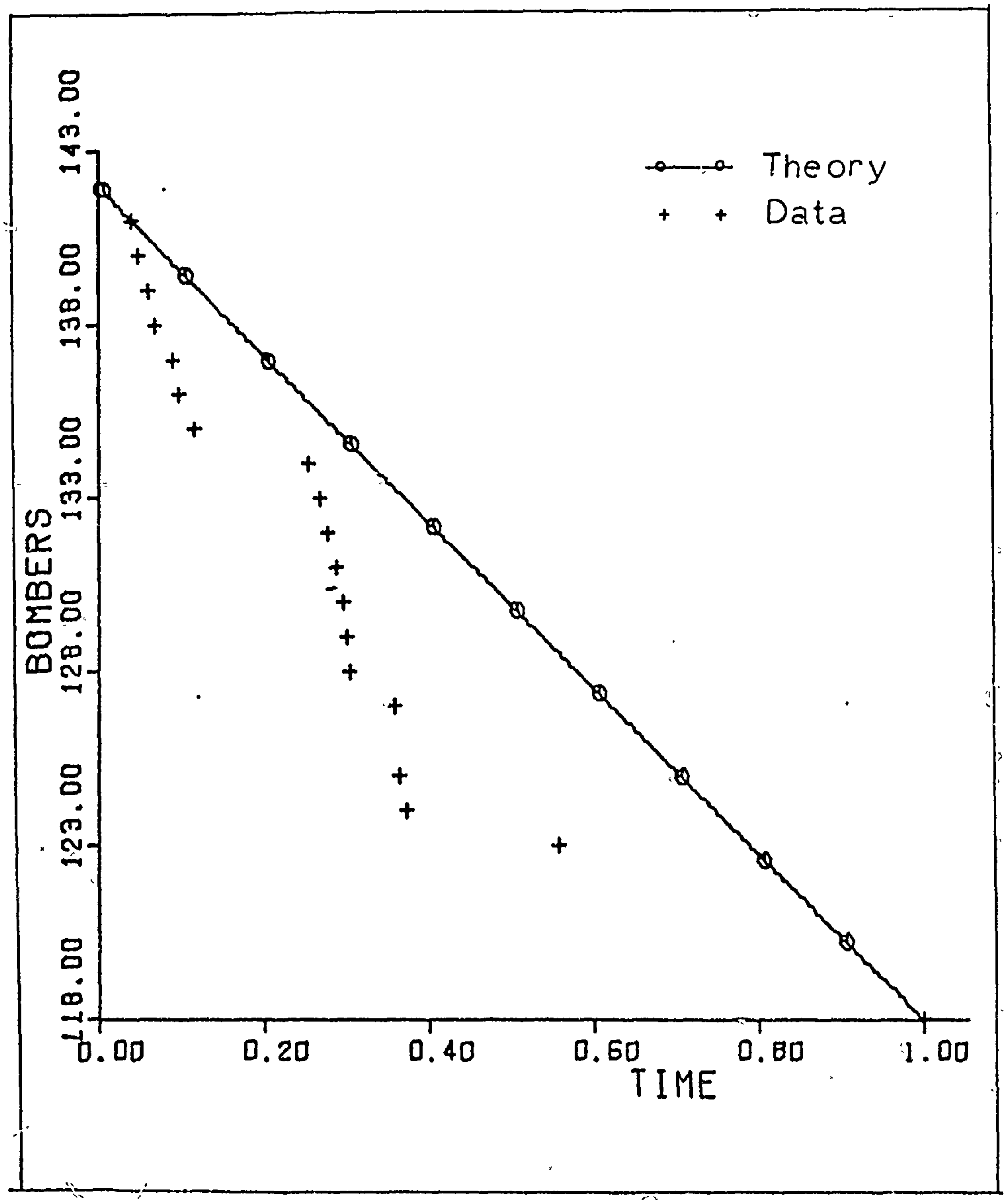

Figure 2 Linear Model Predictions; Regensburg Battle

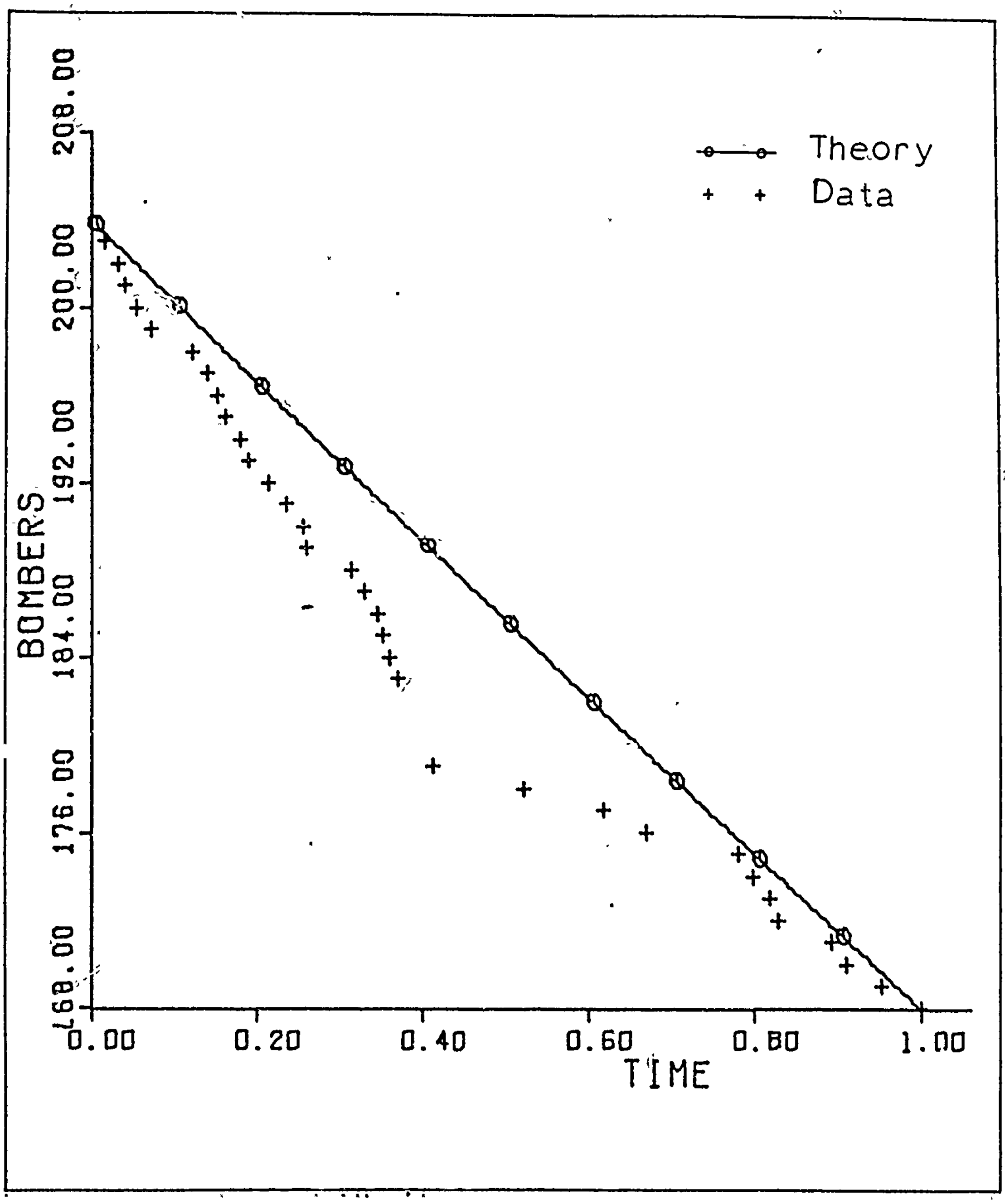

Figure 3 Linear Model Predictions; Schweinfurt Battle

For the Regensburg mission, the greatest percentage error is 7.26%. Nine tenths of the percentage errors are less than 5.18%. For the Schweinfurt mission, 5.57% is the largest percentage error and the ninety percent bound is 3.73%. These errors are, of course, too large to be accepted under the 2% maximum error criterion which was adopted.

The disparity between the shape of the curve and the pattern of the data points is not encouraging for this model. One could achieve nearly the same results by assuming that bomber losses occur in a manner which could be represented by a linear function. Furthermore, since Phillip Morse (Ref 17) describes Lanchester predictions as the most probabilistic outcome, one would expect the curve to lie above some data points and below others. The fact that predictions are consistently larger than historical force sizes may be attributed to the concept of forcing the curve to pass through the initial and terminal values and the magnitude of the value of B. This may be seen by noting the slope of the bomber curve.

$$\dot{m} = -n_o I e^{-Bt} \tag{41}$$

Changes in the slope are dependent on the size of the product Bt. When the value of B is small, .034 for Regensburg and .053 for Schweinfurt, the change in slope from the beginning of the battle, $-n_o I$, to the end of the battle, $-n_o I/e^B$, is also quite small. One would expect that the

errors could be reduced if only the convexity of the bomber curve were more pronounced. As will be seen in the following section, this improvement may be attained by increasing the value of B and shall be acceptable provided that the increase may be plausibly explained.

The knowledge that the predicted values are larger than the actual values may be combined with observations of the size of the errors to allow a simple correction. Since the errors seem to be greatest for values of t between two and seven tenths, a procedure of subtracting a constant from the predicted values in this range would serve to reduce the errors.

An alternative method which may improve the results of the Linear Model would be to rely on some theoretical method involving weapon characteristics to estimate values of the B and I parameters. This approach does not ensure agreement between predicted and historical terminal values and is dependent upon the accuracy of whatever algorithm is used to compute I and B. This approach was not used in this study because an algorithm which is generally accepted yet unclassified is unavailable.

The Claims Model

An alternate method for estimating the exchange rate involves using the number of fighters which are claimed to have been destroyed by bomber aircrews. The resulting model differs from the Linear Model only in the values of

the parameters and shall be referred to as the Claims Model. Claimed fighter kills were invariably larger than the number of the fighters which was recorded as being actually destroyed. In the excitement of battle, American flyers frequently perceived inflated results, while in the aftermath, German officials often de-emphasized losses for propaganda purposes. It is obvious that not every claim was the result of a fighter being destroyed. However, claim numbers are considered to reflect a battle characteristic which has an effect on the combat loss rates. Suppose that no claim was made unless the fighter in question terminated its attack. Then each termination claim implies one of four events. (1) The fighter sought another target. (2) The fighter returned to an air base and recycled. (3) The fighter returned to an air base and was unable to recycle. (4) The fighter crashed or executed a forced landing at a remote site. While the first event has little effect on fighter activity, the other three represent times during which the individual fighter's rate of fire is zero. In this light, claim numbers reflect the effectiveness which bombers have in repelling attackers and whenever a fighter has been repelled, its effectiveness is decreased since it cannot be actively contributing to the objective of destroying bombers. It is possible that, in order to defend themselves, bombers need not destroy their attackers but merely drive them away.

Mathematically, to consider claimed kills as fighter losses has the effect of decreasing E to a value less than one and of increasing the value of B. The B values for Regensburg and Schweinfurt are now .218 and .404 respectively. Claims Model results for these missions are depicted in Figures 4 and 5. For the Regensburg mission, the greatest percentage error is 6.84% and the ninety percent bound is 4.80%. The corresponding values for Schweinfurt are 4.57% and 2.79%. The Claims Model reduces the sum of the absolute values of errors by approximately 8.5% for Regensburg and by 32% for Schweinfurt.

The increased values of B have caused the bomber curves to become more convex and have reduced the magnitude of the errors. The error sum reduction for the Schweinfurt battle is considered to be quite good, but, since there are only two data sets, this improvement may just as easily be the exception as the rule. However, the 8.5% reduction for the Regensburg case is felt to be large enough to justify the use of the Claims Model in preference to the Linear Model. These results are still less accurate than is desired and the majority of the errors result from predictions which are larger than historical values.

The Modified Model

There are two simplifications inherent in a model which assigns a constant value to the combat loss rates. First, it is assumed that effectiveness does not vary

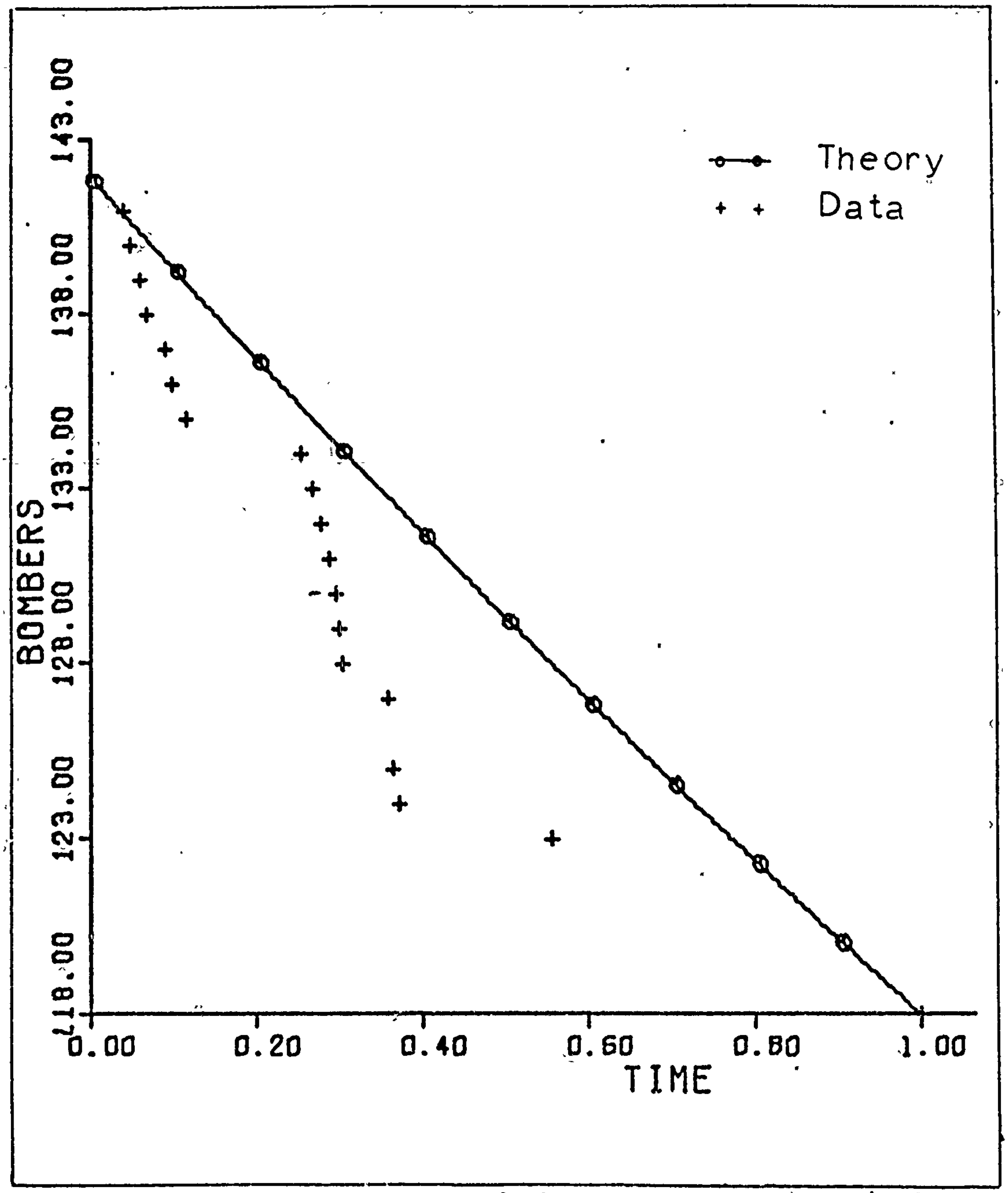

Figure 4 Claims Model Predictions; Regensburg Battle

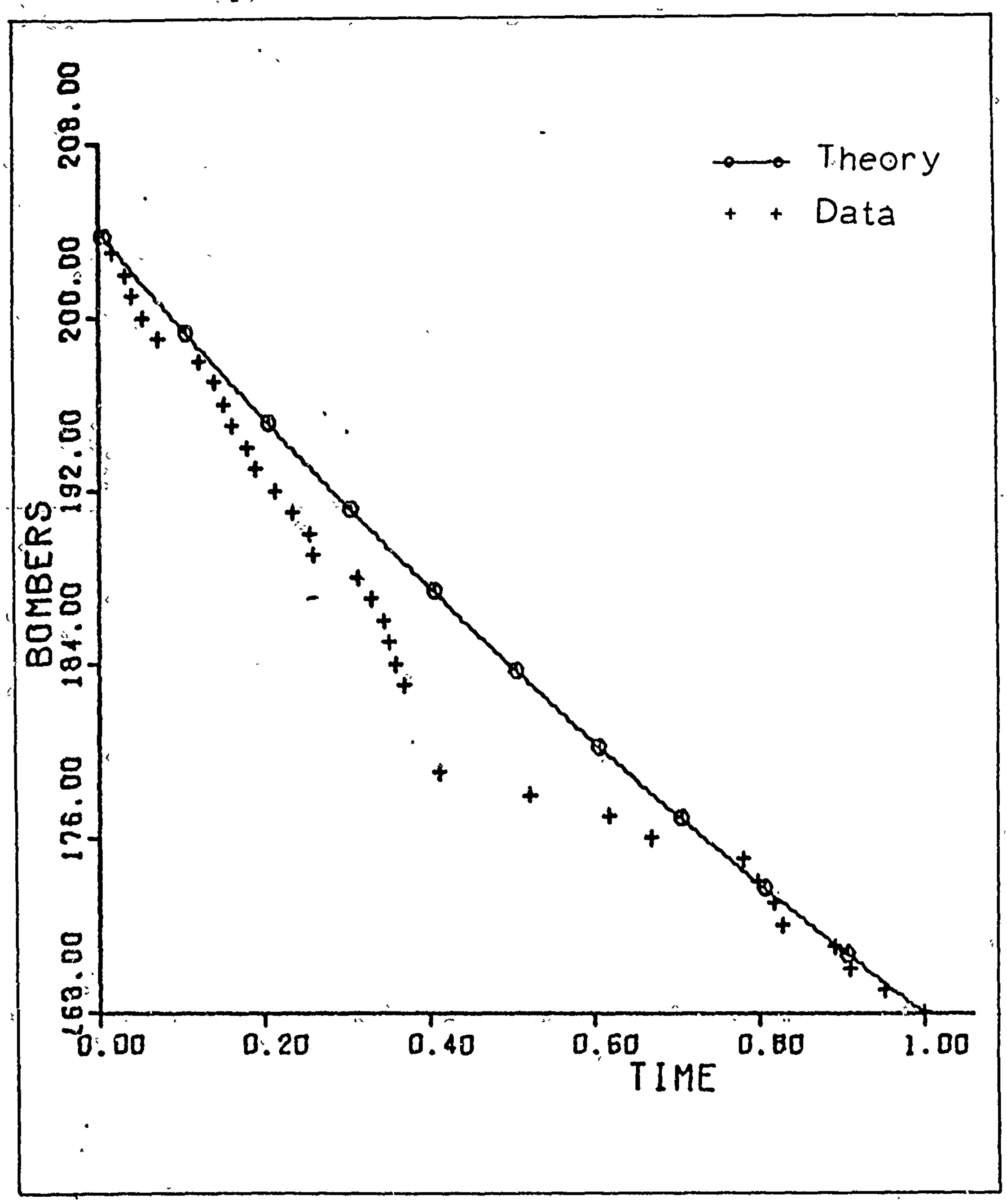

Figure 5 Claims Model Predictions; Schweinfurt Battle

among individual force elements. This is obviously not the case. Consider fighter pilots and their aircraft. The pilot's courage, experience and skill combine with some of the mechanical traits of the machine to determine the accuracy of fire. The rate of fire is dependent upon the pilot's aggressiveness and upon the mechanical condition of his equipment. The assumption which is frequently employed is that effectiveness is randomly distributed among force elements and that the combat loss rate may be equated to the mean value of this distribution. S. Bonder (Ref 5) and C. B. Barfoot (Ref 3) have each generated a distribution for combat loss rates. Both efforts involve discrete distributions which are derived by considering the battle as a stochastic process.

A second simplification assumes that loss rates do not vary with time. This is tantamount to claiming that each element fires its weapons at a constant rate and with constant accuracy throughout the battle. The point is that assigning constant values to the loss rate parameters I and B could be an assumption which is too restrictive. Granted that a model is a simplification of reality, but there must exist some fulcrum at which point the model becomes over simplified and cannot accurately recreate the events which are being studied. It therefore might be beneficial to alter the Claims Model by allowing the I and B values to vary with time or among fighting elements. Varying loss rates among aircraft would complicate a Lanchester model

excessively. This variation would seem to be more appropriate for use with computer simulations which monitor the actions of individual fighting elements. However, consideration of loss rates I and B which vary according to time is well within the scope of this study. The Modified Model which will now be developed is a variation of the Claims Model with non-constant parameters, and is included to illustrate that predictions may be improved by loosening the restrictions which are imposed by the assumptions discussed above.

To derive numerical predictions from a model with variable loss rates, I and B, two questions need to be answered. How do these rates vary as time advances and how do they vary with respect to one another? If the model is to have any general applicability, then it must be supposed that the parameters vary according to some pattern for all strategic bomber air battles. Recall that Frick's original model assumed that both fighter and bomber losses were dependent on the actions of the fighter aircraft, i.e., the frequency with which fighters made attacks on the bombers. Thus, if the bomber loss rate I varies because fighter pilots choose to be more or less aggressive, it is not illogical to assume that the return fire from the bombers and, hence the parameter B, varies in a like manner. Therefore, it is assumed that both parameters vary in the same manner.

To describe the pattern with which these parameters vary with time requires another assumption. The scatter diagrams for the Regensburg and Schweinfurt battles each contain a portion which is steeper than the remainder of the graphs. These portions, occurring when t is near .35 for Schweinfurt and .30 for Regensburg, represent times when bomber losses occurred more frequently than at other times during the battle. It is not unreasonable to suppose that these periods of rapid losses were the result of an intensified effort on the part of fighter pilots. Based on this observation, the loss rates may be assumed to increase in magnitude from the beginning of the battle until some time when the fighting is most intense and a maximum rate is achieved. Following this period of intensity, loss rates decline in magnitude until the end of the battle.

The pattern just described may be represented graphically by a concave, unimodal function of time which achieves a maximum value for a time t which corresponds to the time at which the fighting is most aggressive. Such a curve is presented in Figure 6. Aspects of the curve which are of interest are the value of the local maximum, the time, T, at which this maximum is achieved, and the value when time equals zero. The Modified Model was constructed by approximating this type of function with a bell-shaped curve which may be represented by a normal probability distribution and then replacing the constant

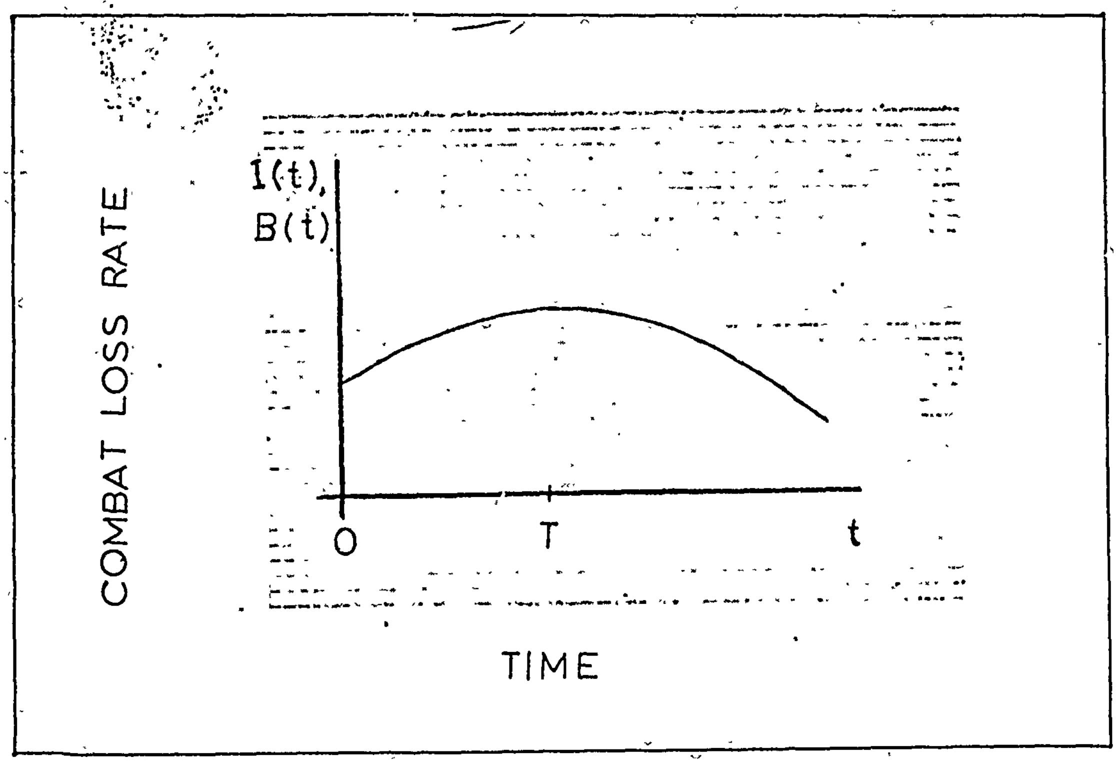

Figure 6 Example of a Variable Combat Loss Rate

parameters I and B which appear in the Claims Model with this function. The bomber loss rate was altered from I to become $I(t) = I \cdot f(t;u,s^2)$ and the fighter loss rate to become $B(t) = B \cdot f(t;u,s^2)$ where it is understood that these functions have a value of zero for negative values of t.

Recall that a combat loss rate may be interpreted as the product of a rate of fire and a single-shot kill probability. It is not unreasonable to suppose that the rate of fire reaches a maximum value near the target. This may be explained by considering that the emotions of the fighter pilots reach a crest when it becomes evident that bombs will actually be dropped on their territory. Under this assumption, the mean value, u, of the loss rate functions

is considered to be the time, T, that the force was over the target.

In order to determine the value of the curve for times zero and T, it was assumed that at the beginning of the battle the proficiency of the aircrews' ability to shoot down enemy planes jumps from zero (because no rounds were being fired prior to this time) to some level which is a percentage of the maximum value that is achieved when the action becomes most intense. That is, the training and experience of the airmen involved allows them to begin shooting with some accuracy without any warming up. In all probability, the rate of fire at this early stage is not maximum as the fighters exert some effort and time to "feel out" the invaders. By studying the respective scatter diagrams, it may be noted that the frequency of losses, i.e., the slope of a line connecting the data points, near the beginning of the battle is approximately seven tenths of the frequency during the most intense portions of the battle. This relationship appears in both battles and is interpreted to imply that the loss rates at t equal to zero and t equal to T would be of this same ratio. That is, the ratios of $I(0)$ to $I(T)$ and $B(0)$ to $B(T)$ should each equal seven tenths. This ratio provides a means of determining the value of s. Using the value of a normal distribution and knowing the value of the mean allows one to assign a value to s which ensures that

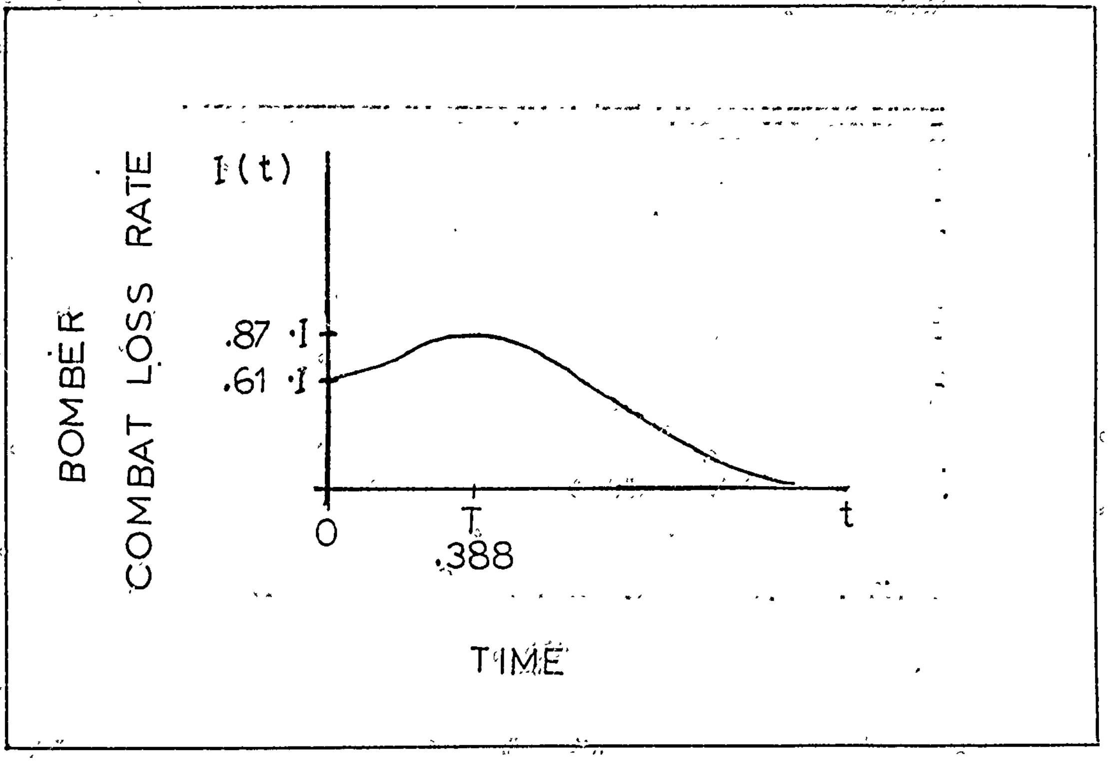

Figure 7 Bomber Combat Loss Rate for Modified Model; Schweinfurt Battle

the ratio of f(o) to f(u) is equal to seven tenths. The resulting function for I(t) in the Schweinfurt battle is shown in Figure 7. For this particular example, the time over target T is equal to .388 and the relationship f(0) = .7f(T) is desired. Equation (42) may be solved to determine the correct value of s.

$$\frac{.7}{\sqrt{2\pi}\,s} = \frac{1}{\sqrt{2\pi}\,s}\, e^{-\frac{1}{2}(-u/s)^2} \tag{42}$$

The result in this case is that s is approximately .459.

The set of Lanchester equations which represent the Modified Model are

$$\dot{m} = -I(t)\,n \tag{43}$$

$$\dot{n} = -B(t)\,n \tag{44}$$

Equation (44) may be solved and results in

$$\bar{n} = n_o e^{-B(F(t)-F(0))} \tag{45}$$

where $F(t) = \int_{-\infty}^{t} f(x;u,s^2)dx$. Substituting this value for n in equation (43) and integrating yields the desired bomber curve expression.

$$m = m_o - \frac{I}{B} n_o\left[1 - e^{-B(F(t)-F(0))}\right] \tag{46}$$

Results of the Modified Model are depicted in Figures 8 and 9. The greatest percentage error for Regensburg is 5.67%. The ninety percent bound is 4.03%. The sum of absolute values of errors recorded for this mission with the Claims Model is reduced by approximately 18%. Corresponding values for the Schweinfurt mission are 3.72%, 1.91% and 9%.

The reductions in the error sums and the consistent improvements in percentage errors are felt to be significant enough to warrant the application of the Modified Model. These results support the view that variances of the loss rate parameters do have an effect on bomber losses which needs to be incorporated in the model. Although the assumption of constant values for I and B is a reasonable first approximation, the Modified Model predictions for the Regensburg and Schweinfurt battles suggest that this assumption is, perhaps, too restrictive. Reduction of

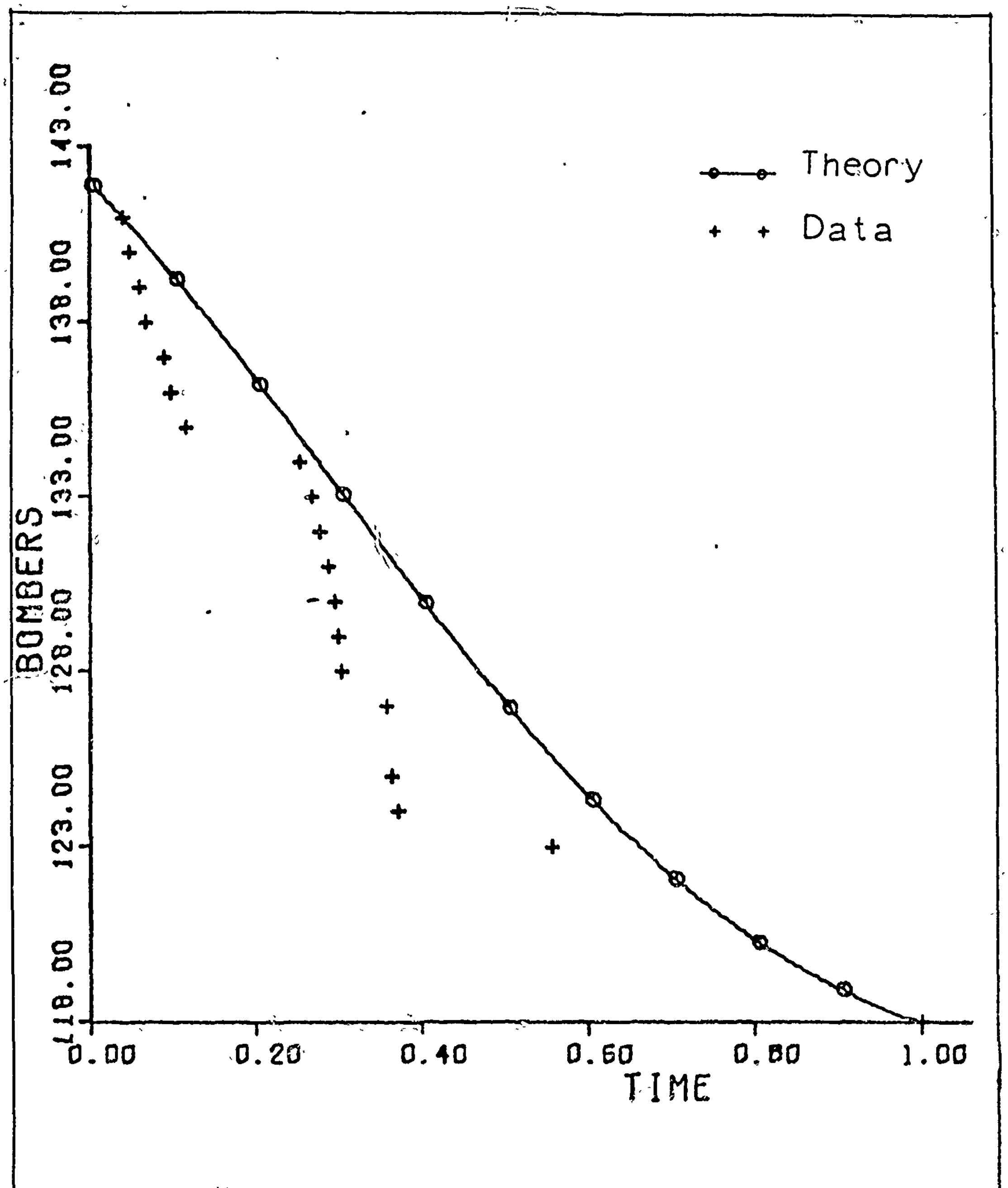

Figure 8 Modified Model Predictions; Regensburg Battle

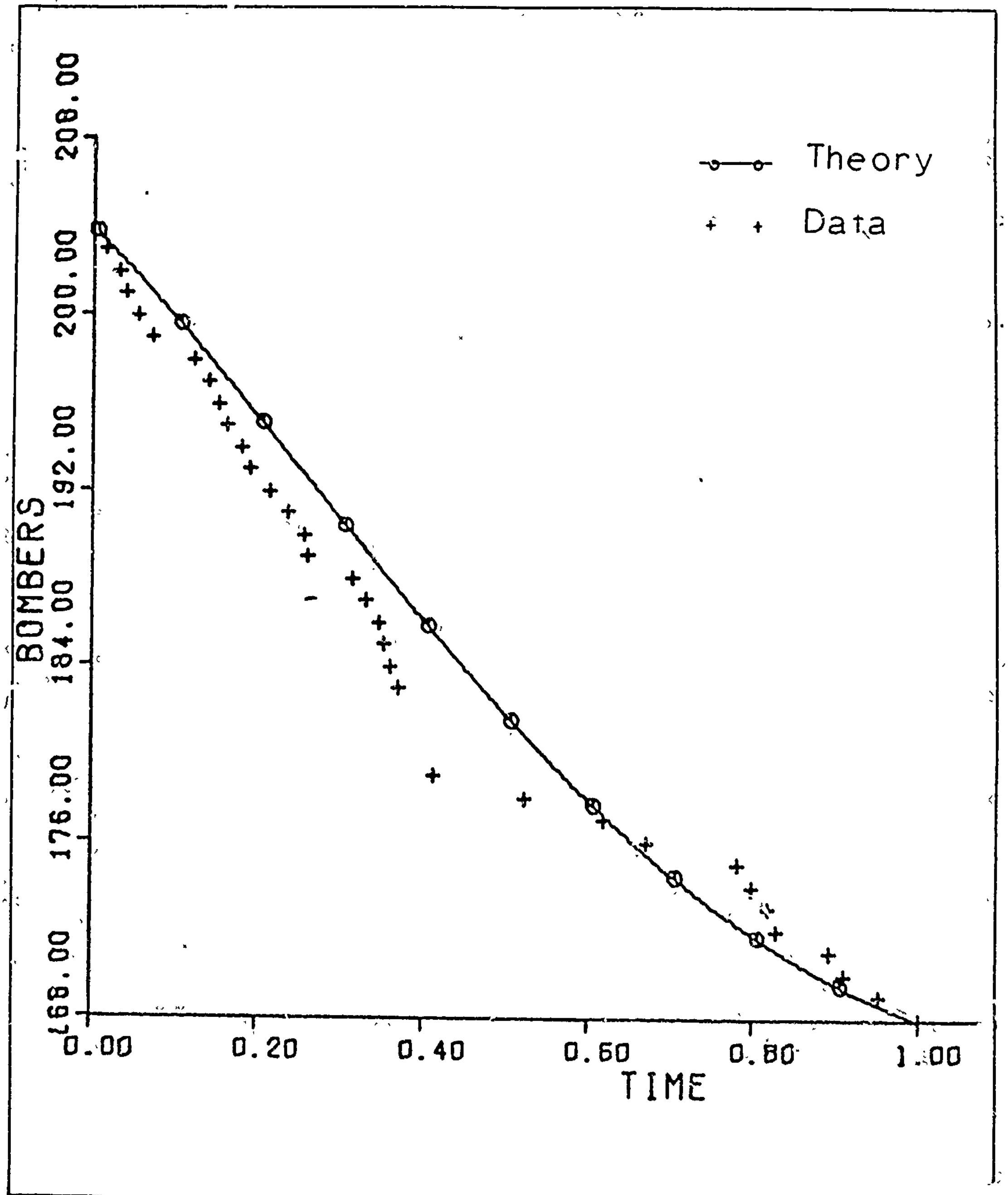

Figure 9 Modified Model Predictions;
Schweinfurt Battle

error sums of less than five percent would have been deemed insignificant.

It should be noted that additional improvements may be obtained by assigning a value to the mean of the I(t) and B(t) functions which corresponds to the period of the battle during which bomber losses occurred with greatest frequency. This also results in predictions falling both above and below historical data values. Such times, t = .29 for Regensburg or .35 for Schweinfurt, occur near the time over target but do not fit into any convenient pattern for the two battles. As models are valued for their predictive ability, the times over target, which would be available prior to the actual flying of a mission, are used in this research rather than the times of greatest activity, which would not be known prior to the battle.

It may be that a more refined method of describing the changes in loss rate magnitudes could result in exceptionally accurate predictions with the Modified Model. For instance, it may be that the functions I(t) and B(t) should be bimodal about the time over target or even cyclic throughout the battle. On the other hand, variables which are relevant to the study of air battles may be such that the predictions of a deterministic model are of limited accuracy.

The Ploesti Model

Recall that the data for the Ploesti raid revealed that only twenty percent or ten bomber casualties could be attributed to the efforts of fighter aircraft. This, combined with the fact that the majority of bomber losses occurred at the target area during a brief time interval rather than across the countryside during a running air battle cause this experience to differ from the more orthodox missions flown against Regensburg and Schweinfurt. If either the Linear or Claims Model is applied to the Rumanian battle, errors become so large that the predictions cannot be considered as being accurate. A model variation was sought which might improve on these outcomes. The variation which was adopted was taken from M. B. Schaffer's study on guerrilla engagements (Ref 21). Schaffer's work concerns a force which may use the aid of supporting weapons systems during a portion of the battle. The analogy between this and the air battle is that the ground fire which the Ploesti bombers encountered at the target area may assume the role of a supporting weapons system.

Schaffer asserts that the effect of several types of supporting fire is additive. The changes in bomber force size, which are attributed to fire from ground weapons, may be relected by adding an expression for ground fire strength and effectiveness to the Linear Model. Let g be a function of time which represents the strength of the

Ploesti ground forces. Let A represent the combat loss rate for bombers when being attacked by ground artillery. The resulting set of differential equations is referred to as the Ploesti Model.

$$\dot{m} = -I\,n - Ag \tag{47}$$

$$\dot{n} = -Bn \tag{48}$$

It is not possible to develop an expression for the bomber force size from equations (47) and (48) without more information on the function g. Little has been recorded concerning ground fire activity during the Ploesti raid but it is felt that its strength over time may be easily approximated. Ground fire makes no direct battle contribution except while targets are within range. The activity at a typical anti-aircraft battery may be envisioned as follows. The men are aware that an invader force is inbound and await anxiously for the bombers to appear. The bombers pass within range and the order to fire is given. Once the first round is fired, the remaining guns join the battle after a very brief lapse of time. Fire is continued at a maximum rate until the bombers pass out of range and the guns are silenced while the men await their next target. This scenario may be represented graphically as the square wave pattern shown in Figure 10. The amplitude of the wave, representing the full strength of the battery, diminishes only slightly from cycle to cycle. The wave

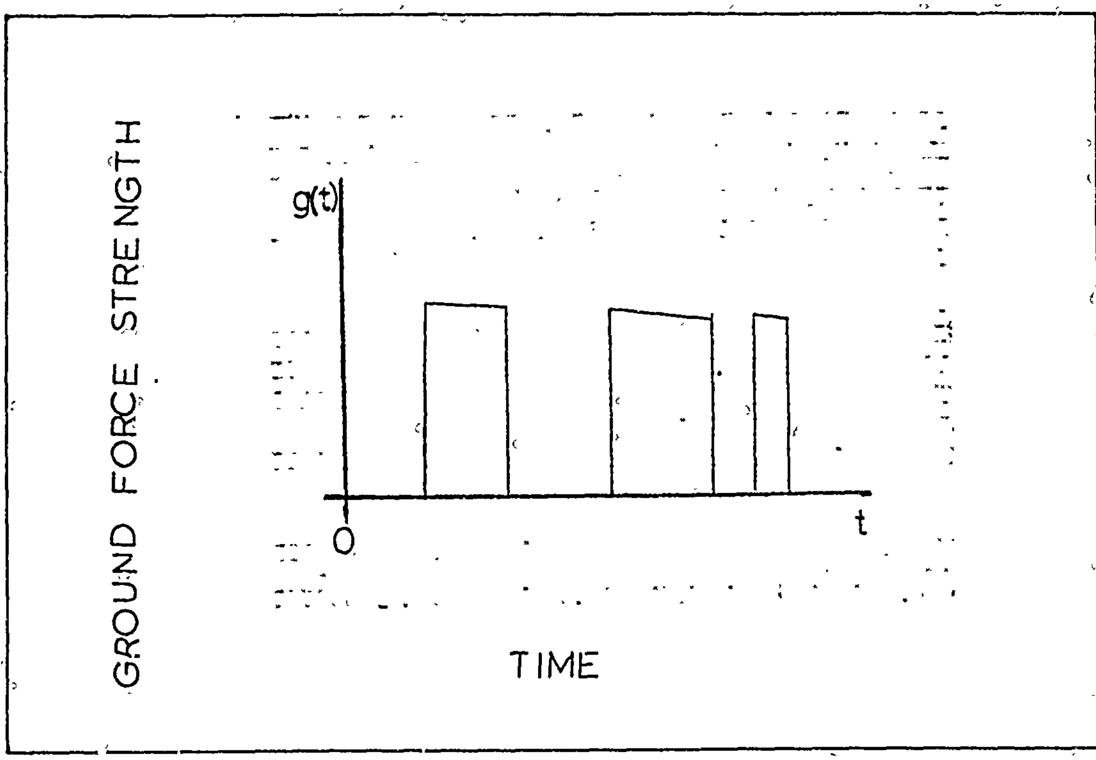

Figure 10 An Approximation of Ground Force Strength

length and the crest width are determined by the times when targets pass within range and the time periods during which targets remain in range. Considering the ground forces which surrounded Ploesti to be one large battery, this battle contained only one cycle of anti-aircraft fire.

The square wave which may be used to represent the ground force strength may be approximated by a continuous bell-shaped curve. The function g may now be defined to be zero for values of t less than or equal to zero and equal to $f(t;u,s^2)$ for values of t greater than zero. The function $f(t;u,s^2)$ represents a normal probability distribution with mean u and variance s^2. It was chosen because the shape of this curve may be easily changed by adjusting

the parameters u and s. The value of the mean is assumed to be the median between the first bomber group's time over target, denoted by t', and the last group's time over target, denoted by t''. Fifteen minutes is added to t'' to account for stragglers and allow the bombers time to clear the area. The standard deviation s, was then determined to allow t'' plus fifteen minutes to lie three standard deviations from the mean. This procedure ensures that 99% of the area beneath the curve lies between time values during which the bomber fleet was known to be over the battery. This approximation is represented in Figure 11.

Accepting this procedure and its approximation for g, allows a bomber function to be determined. Letting $F(t)$ equal the integral $\int_{-\infty}^{t} f(x;u,s^2)dx$, equation (49) shows the bomber force size function for the Ploesti Model. Equation (49) is derived in a manner similar to that used for the Linear Model. Equation (48) is solved as a linear differential equation and the resulting expression for n may then be substituted in equation (47) and integration produces the desired expression for m.

$$m = m_o - \frac{I}{B} n_o(1 - e^{-Bt}) - A(F(t) - F(0)) \tag{49}$$

Defining the function g in this manner implies that ground fire has no effect on the bombers prior to the battles beginning, that there is some psychological effect which may alter aircrew performance as the men anticipate the

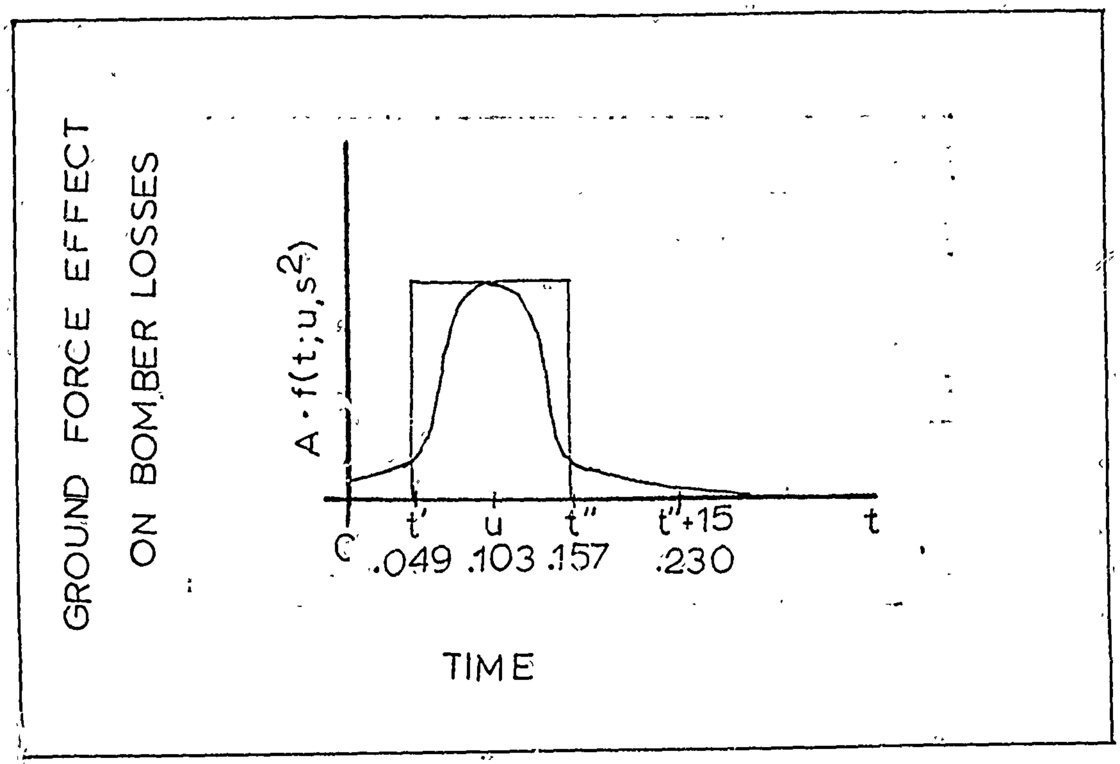

Figure 11 A Representation of the Ground Force Effect on Bomber Losses at Ploesti

ground fire, that the effect is most intense while bombers are over anti-aircraft installations and that there is the lingering effect of damaged aircraft which may be forced to land after leaving the area.

Values for the parameters I and B are determined by using the same method as was used for the Linear Model. Only bomber losses which can be attributed to interceptor actions are used in these computations. This implies the assumption that the air battle and the ground-to-air battle can be considered as separate entities. This assumption is not too different from reality. Ploesti defense plans stipulated that German fighters would not enter air space

which was assigned to anti-aircraft batteries. A value is then assigned to the parameter A which forces the curve to match the data when t equals one. This value may be determined by setting t equal to one in equation (49) and solving for A.

$$A = \frac{m(1) - m_o + E n_o (1 - e^{-B})}{F(0) - F(1)} \tag{50}$$

Comparison results are graphed in Figure 12 with the largest percentage error being 3.21%. Though the errors are greater than 2%, the results are felt to be good considering the coarse values which were assigned to u and s. It is not unreasonable to conclude that the effect of ground fire is directly related to the time that a bomber force flies within range of the anti-aircraft weapons. By adding one term to the bomber force differential equation, it is possible to account for the battle variations which high altitude bombers encountered when they came down to tree top level. The main motivation for including the Ploesti Model in this research is to demonstrate the flexibility of the Linear Model. A second comparison which uses claimed fighter kills in the computations produces similar results in which the error sum is reduced by nine percent. The graph of this second prediction is not included since the curve strongly resembles the one in Figure 12.

A summary of all of the results is presented in Table I.

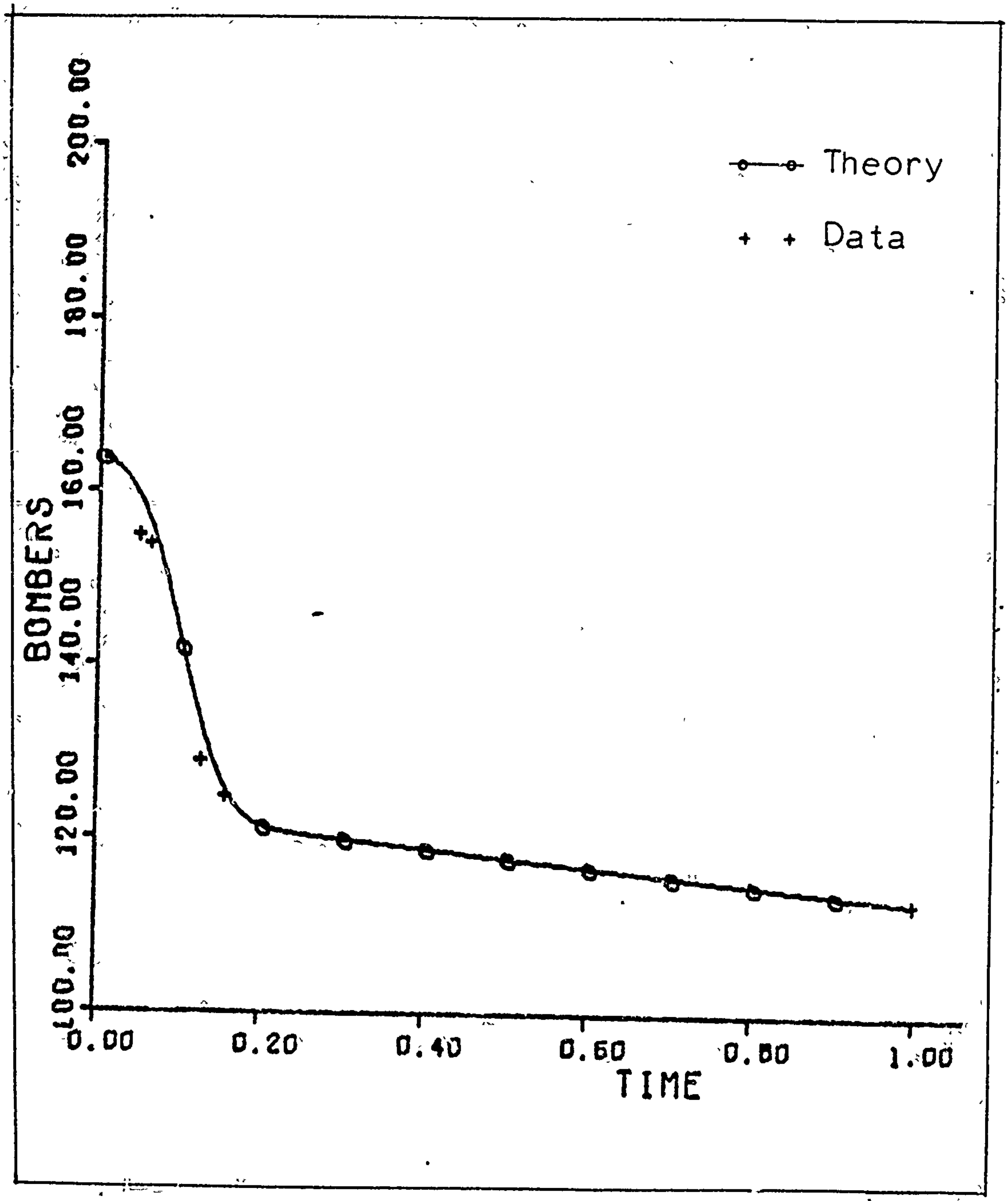

Figure 12 Ploesti Model Predictions; Ploesti Battle

TABLE I

SUMMARY OF RESULTS

MODEL AND BATTLE	GREATEST PERCENTAGE ERROR	90% BOUND	ERROR SUM	PER CENT REDUCTION OF ERROR SUM		
				OVER LINEAR	OVER CLAIMS	OVER PLOESTI
Linear Regensburg	7.26	5.18	73.32			
Claims Regensburg	6.84	4.80	66.99	8.65		
Modified Regensburg	5.67	4.03	54.73	25.2	18.3	
Linear Schweinfurt	5.57	3.73	110.67			
Claims Schweinfurt	4.57	2.79	75.38	31.9		
Modified Schweinfurt	3.72	1.91	68.24	38.4	9.48	
Ploesti	3.21	3.21	11.52			
Ploesti (Claims)	2.95	2.95	10.48			9.05

VI. Conclusions and Recommendations

This research may be considered a qualified success in that it adds to the list of existing historical validations. The improvements which are indicated by the reduction of error magnitudes when the Claims and Modified Models are used are encouraging. The definition of a model which was used indicated that success might depend on the correct identification of "features of the situation relevant to the questions being studied." These noted improvements reflect the plausibility that, in the study of air battles, a bomber's ability to repel attacking fighters and variable combat loss rates are relevant features which should be included. There is some disappointment because none of the model predictions met the adopted criterion. However, it should be emphasized that the selection of a 2% maximum percentage error is an arbitrary choice and not necessarily the correct value. Verifications of terrestrial conflicts will be mentioned which do meet this criterion but this is not to imply that identical standards should apply to both land and air battles. It should be noted that the ninety per cent bound for the Modified Model does meet this self-imposed measure of quality and that there is no indication that further improvements cannot be made. Perhaps a different method of evaluating parameters or selection of another relevant factor is in order. The outcome of this one

effort does not imply that the models are necessarily invalidated.

It is of interest to compare the results of this research with the two similar studies by Engel and Fain. Engel was well justified in claiming that the Lanchester equations which he employed produced a good re-enactment of the battle of Iwo Jima. All of his percentage errors were less than one. Fain also felt justified in concluding that the historical data did not contradict his model predictions. His percentage errors were observed to be less than two. The failure of this research to achieve results which are comparable to the results of these other studies may be attributed to either procedural errors or to the possibility that the nature of air battles is such that the accuracy which was experienced in applying Lanchester's theory to infantry engagements is not attainable when the theory is applied to aerial conflicts.

Possible extensions of this effort include alternative methods of determining the combat loss rate parameters. These I and B values might be estimated by forcing the bomber fleet predictions to match the data at some point other than at the end of the battle. I and B might be determined externally, using an algorithm based on weapons systems characteristics, and then inserted in the model. Various functions might be used to represent I(t) and B(t) in the Modified Model. A variation of this study might be an attempt to consider attrition rates of battles

which involve heterogeneous forces, i.e., bombers accompanied by fighter escorts.

Bibliography

1. Army Air Forces Statistical Digest-World War II. A report prepared by the Office of Statistical Control, December, 1945.

2. Bach, R. E., Dolansky, L., and Stubbs, H. L. "Some Recent Contributions to the Lanchester Theory of Combat." Operations Research, Vol X, pp 314-326. 1962.

3. Barfoot, C. B. "Comment on Seth Bonder Paper." Operations Research, Vol XVII, pp 888-895. 1969.

4. Baumbach, Werner. Life and Death of the Luftwaffe. New York: Coward-McCann, Inc., 1949.

5. Bonder, Seth, "The Lanchester Attrition Rate Coefficient." Operations Research, Vol XV, pp 221-232. 1967.

6. Craven, Frank Wesley, and Cate, James Lea (eds.). The Army Air Forces in World War II. Vol II. Europe - Torch To Pointblank. Chicago: The University of Chicago Press, 1949.

7. Dolansky, Ladislav. "Present State of the Lanchester Theory of Combat." Operations Research, Vol XII, pp 344-358. 1964.

8. Dugan, James, and Steward, Carroll. Ploesti. New York: Random House, 1962.

9. Engel, Joseph H. "A Verification of Lanchester's Law." Operations Research, Vol II, pp 163-171. 1954.

10. Fain, William W., et. al. "Validation of Combat Models Against Historical Data." Consolidated Analysis Center, Inc., Arlington, Virginia. April, 1970.

11. Freeman, Roger Anthony. The Mighty Eighth--Units, Men and Machines. London: MacDonald and Company (Publishers) Ltd., 1970.

12. Frick, Roy Kimble. "Interaction of Forces as Discrete Processes with Application to Air Battle Analysis." Deputy for Development Planning, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio. October, 1970.

13. Gurney, Gene (ed.). Great Air Battles. New York: Franklin Watts, Inc., 1963.

14. Helmbold, Robert L. "Air Battles and Ground Battles--A Common Pattern?" The RAND Corporation, Santa Monica, California, P-4548. January, 1971.

15. Helmbold, Robert R. "Lanchester Parameters for Some Battles of the Last Two Hundred Years." Tech Ops, Combat Operations Research Group. February, 1961.

16. Helmbold, Robert L. "Some Observations on the Use of Lanchester's Theory for Prediction." Operations Research, Vol XII, pp 778-781. 1964.

17. Morse, Phillip M., and Kimball, George E. Methods of Operations Research. New York: The Technology Press of Massachusetts Institute of Technology, 1951.

18. Newman, J. R. The World of Mathematics, Vol IV. New York: Simon & Shuster, 1956.

19. The Ploesti Mission; 1 August 1943. (Short Title: AAFRH-3). A report prepared by the Assistant Chief of Staff, Intelligence, Historical Division, June, 1944.

20. Rust, Kenn C. The 9th Air Force in World War II. Fallbrook, California: Aero Publishers, Inc., 1967.

21. Schaffer, M. B. "Lanchester Models of Guerrilla Engagements." Operations Research, Vol XVI, pp 457-488. 1968.

22. Stormont, Capt John W., The Combined Bomber Offensive; April - December, 1943. (Short Title: AAFRH-19). A report prepared for the Army Air Force Historical Office, 1945.

23. Sunderman, James F. (ed.). World War II in the Air; Europe. New York: Bramhall House, 1963.

24. Sweetman, John. Schweinfurt - Disaster in the Skies. New York: Ballantine Books, Inc., 1971.

25. "A Syllabus of Equations for Force Effectiveness Analysis." United States Air Force, Assistant Chief of Staff, Studies and Analysis. 1 December 1970.

26. Wallis, P. R. "Recent Developments in Lanchester Theory." Operational Research Quarterly, Vol XIX, pp 191-195. 1968.

27. Weiss, Herbert K. "Combat Models and Historical Data; The U. S. Civil War." Operations Research, Vol XIV, pp 759-790. 1966.

28. Willard, D. "Lanchester as Force in History; An Analysis of Land Battles of the Years 1618-1905." RAC-TP-74. Research Analysis Corporation, Bethesda, Maryland. November, 1962.

29. Wolff, Leon. Low Level Mission. Garden City, New York: Doubleday and Co., Inc., 1957.

APPENDIX

TABLE II

REGENSBURG DATA AND PARAMETERS

BOMBER FORCE SIZE	CUMULATIVE NORMALIZED TIME	PERCENTAGE ERROR LINEAR	CLAIMS	MOD.
141	.039	.03	.03	.02
140	.048	.59	.52	.57
139	.060	1.11	1.01	1.06
138	.067	1.71	1.61	1.66
137	.090	2.05	1.91	1.94
136	.097	2.67	2.53	2.55
135	.116	3.09	2.92	2.91
134	.254	1.36	1.04	.55
133	.267	1.89	1.56	1.00
132	.278	2.46	2.12	1.50
131	.287	3.08	2.72	2.06
130	.295	3.72	3.36	2.64
129	.299	4.45	4.08	3.34
128	.304	5.17	4.80	4.03
127	.357	4.99	4.59	3.52
125	.363	6.56	6.14	5.[illegible]3
124	.371	7.26	6.84	5.67
123	.555	4.54	4.09	2.14
118	1.000	--	--	--
		B = .034	.219	.296
		1/B = 2.4	.407	.407

m_o = 142
m(1) = 118
n_o = 300
n(1) = 290
n(1) = 241 (claims)
Time over Target = .353

TABLE III

SCHWEINFURT DATA AND PARAMETERS

BOMBER FORCE SIZE	CUMULATIVE NORMALIZED TIME	PERCENTAGE ERROR		
		LINEAR	CLAIMS	MOD.
203	.016	.20	.14	.19
202	.031	.42	.31	.39
201	.039	.78	.63	.73
200	.053	1.02	.82	.94
199	.072	1.18	.91	1.05
198	.121	.78	.36	.50
197	.140	.94	.46	.58
196	.151	1.25	.74	.84
195	.162	1.56	1.02	1.10
194	.179	1.76	1.18	1.22
193	.190	2.08	1.46	1.49
192	.213	2.17	1.50	1.46
191	.236	2.27	1.55	1.43
190	.255	2.44	1.68	1.50
189	.260	2.89	2.11	1.91
188	.313	2.41	1.54	1.14
187	.330	2.62	1.74	1.26
186	.345	2.88	1.97	1.43
185	.352	3.30	2.38	1.80
184	.359	3.73	2.79	2.18
183	.369	4.09	3.14	2.49
179	.411	5.57	4.57	3.72
178	.521	3.94	2.92	1.67
177	.617	2.58	1.63	.19
176	.670	2.08	1.19	.26
175	.779	.45	.23	1.50
174	.797	.66	.02	1.20
173	.818	.82	.22	.92
172	.827	1.22	.64	.46
171	.891	.49	.10	.68
170	.909	.70	.37	.30
169	.951	.43	.24	.15
168	1.000	--	--	--
		B = .053	.461	.649
		1/B = 2.4	.404	.404

m_o = 204
$m(1)$ = 168
n_o = 290
n_o = 241 (claims)
$n(1)$ = 275
$n(1)$ = 152 (claims)
Time over Target = .388

TABLE IV

PLOESTI DATA AND PARAMETERS

BOMBER FORCE SIZE	CUMULATIVE NORMALIZED TIME	PERCENTAGE ERROR	
		PLOESTI	CLAIMS
155	.049	2.96	2.87
154	.064	1.45	1.33
129	.128	3.21	2.95
125	.157	.44	.13
113	1.000	--	--
		B = .121	.665
		1/B = .833	.196
		A = 41.316	41.316

$\hat{m}_o$ = 164

m(1) = 113

n_o = 105

n(1) = 93

n(1) = 54 (claims)

Time over Target = .103

Vita

John Henry Latchaw was born on 27 September 1944 in Guthrie, Oklahoma. He graduated from Guthrie High School in 1962, attended Oklahoma State University and then Phillips University, Enid, Oklahoma from which he received the degree of Bachelor of Science in 1966. He attended Officer Candidate School at San Antonio, Texas and received a commission in the United States Air Force on 16 February 1968. After graduating from the Basic Officer Communications Course at Keesler Air Force Base in December, 1968, he served the 916th Air Refueling Squadron at Travis Air Force Base as the Tactical Communications Officer until June, 1970. He was assigned to the Air Force Institute of Technology as a Graduate Systems Analysis student in July, 1970.

Permanent Address: 318 East Cleveland Avenue
Guthrie, Oklahoma 73044

This thesis was typed by Mrs. Anna L. Lloyd.